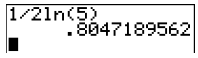
1a $(x^3-27)(x^2-4)=0$
$x^3-27=0 \vee x^2-4=0$
$x^3=27=3^3 \vee x^2=4=2^2$
$x=3 \vee x=2 \vee x=-2.$

1b $5x(x^2-4)=15(x^2-4)$
$x^2-4=0 \vee 5x=15$
$x^2=4=2^2 \vee x=3$
$x=2 \vee x=-2 \vee x=3.$

1c $(x^2-4)^2=(x^2-8)^2$
$x^2-4=x^2-8 \vee x^2-4=-x^2+8$
$0=-4 \vee 2x^2=12$
geen opl. $\vee\ x^2=6$
$x=\sqrt{6} \vee x=-\sqrt{6}.$

2a In de opgave staat $\ln(2x+5)$ en dat is alleen gedefineerd als $2x+5>0$. Daarom moet bij de gevonden waarden gecontroleerd worden of ze aan deze voorwaarde voldoen.

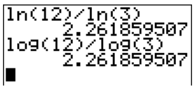
2b $(e^{2x}-5)\cdot\ln(2x-5)=0$ ($2x-5>0 \Rightarrow x>2{,}5$)
$e^{2x}-5=0 \vee \ln(2x-5)=0$
$e^{2x}=5 \vee 2x-5=e^0=1$
$2x=\ln(5) \vee 2x=6$
$x=\frac{1}{2}\ln(5)$ (voldoet niet) $\vee\ x=3$ (voldoet).



3a $(3x^2-5)^2=4x^2$
$3x^2-5=2x \quad\vee\quad 3x^2-5=-2x$
$3x^2-2x-5=0 \quad\vee\quad 3x^2+2x-5=0$
$D=(-2)^2-4\cdot3\cdot-5=64 \qquad D=2^2-4\cdot3\cdot-5=64$
$x=\frac{2-8}{2\cdot3}=\frac{-6}{6}=-1 \vee x=\frac{2+8}{2\cdot3}=\frac{10}{6}=\frac{5}{3} \vee x=\frac{-2-8}{2\cdot3}=\frac{-10}{6}=-\frac{5}{3} \vee x=\frac{-2+8}{2\cdot3}=\frac{6}{6}=1.$

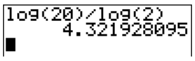
3b $(4x-1)^2=(3x-2)^2$
$4x-1=3x-2 \vee 4x-1=-3x+2$
$x=-1 \vee 7x=3$
$x=-1 \vee x=\frac{3}{7}.$

3c $(3^x-12)\cdot{}^3\log(2x+1)=0$ ($2x+1>0 \Rightarrow x>-0{,}5$)
$3^x-12=0 \vee {}^3\log(2x+1)=0$
$3^x=12 \vee 2x+1=3^0=1$
$x={}^3\log(12)$ (voldoet) $\vee\ 2x=0.$
$x={}^3\log(12) \vee x=0$ (voldoet).



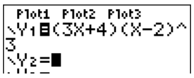
3d $(x-1)\cdot\cos(2x+\frac{1}{4}\pi)=0$
$x-1=0 \vee \cos(2x+\frac{1}{4}\pi)=0$
$x=1 \vee 2x+\frac{1}{4}\pi=\frac{1}{2}\pi+k\cdot\pi$
$x=1 \vee 2x=\frac{1}{4}\pi+k\cdot\pi$
$x=1 \vee x=\frac{1}{8}\pi+k\cdot\frac{1}{2}\pi.$

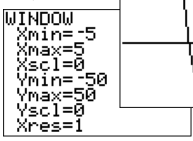
3e $2^x\cdot\log(4x+1)=20\cdot\log(4x+1)$ ($4x+1>0 \Rightarrow x>-0{,}25$)
$\log(4x+1)=0 \vee 2^x=20$
$4x+1=10^0=1 \vee x={}^2\log(20)$ (voldoet)
$4x=0 \vee x={}^2\log(20)$
$x=0$ (voldoet) $\vee\ x={}^2\log(20).$

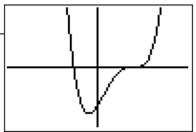


3f $x^3\cdot\sin(2x)=\sin(2x)$
$\sin(2x)=0 \vee x^3=1$
$2x=k\cdot\pi \vee x=1$
$x=k\cdot\frac{1}{2}\pi \vee x=1.$

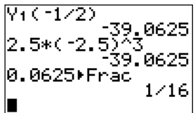
4a $f(x)=0 \Rightarrow (3x+4)(x-2)^3=0$
$3x+4=0 \vee (x-2)^3=0$
$3x=-4 \vee x-2=0$
$x=-\frac{4}{3} \vee x=2.$







4b $f(x)=(3x+4)(x-2)^3 \Rightarrow f'(x)=3\cdot(x-2)^3+(3x+4)\cdot3(x-2)^2\cdot1$
$=3(x-2)\cdot(x-2)^2+3(3x+4)\cdot(x-2)^2$
$=(3x-6+9x+12)\cdot(x-2)^2=(12x+6)(x-2)^2$
$f'(x)=0 \Rightarrow (12x+6)(x-2)^2=0$
$12x+6=0 \vee (x-2)^2=0$
$12x=-6 \vee x-2=0$
$x=-\frac{1}{2} \vee x=2.$
Minimum (zie plot) $f(-\frac{1}{2})=(3\cdot-\frac{1}{2}+4)(-\frac{1}{2}-2)^3=2{,}5\cdot(-2{,}5)^3=-39\frac{1}{16}.$



4c $f(x)=3x+4 \Rightarrow (3x+4)(x-2)^3=3x+4$
$3x+4=0 \vee (x-2)^3=1=1^3$
$3x=-4 \vee x-2=1$
$x=-\frac{4}{3} \vee x=3.$
$f(-\frac{4}{3})=0$ (zie 4a) $\Rightarrow A(-\frac{4}{3},0)$ en
$x=3 \Rightarrow y=3\cdot3+4=13 \Rightarrow B(3,13).$

4d $f(x)=(3x+4)(x-2) \Rightarrow (3x+4)(x-2)^3=(3x+4)(x-2)$
$(3x+4)(x-2)=0 \vee (x-2)^2=1=1^2$
$3x+4=0 \vee x-2=0 \vee (x-2)^2=1=1^2$
$3x=-4 \vee x=2 \vee x-2=1 \vee x-2=-1$
$x=-\frac{4}{3} \vee x=2 \vee x=3 \vee x=1.$
$(-\frac{4}{3},0)$ (zie 4a), $(2,0)$ (zie 4a), $(3,13)$ (zie 43c) en $(1,-7)$.

5a $\frac{2x-1}{x-1}=0$ (teller $=0$ en noemer $\neq 0$) $\Rightarrow 2x-1=0 \Rightarrow 2x=1 \Rightarrow x=\frac{1}{2}$ (voldoet).

5b $\frac{2x-1}{x-1}=\frac{3x+1}{x+1}$ ($x\neq1$ en $x\neq-1$)
$(2x-1)(x+1)=(3x+1)(x-1)$
$2x^2+2x-x-1=3x^2-3x+x-1$
$-x^2+3x=0$
$-x(x-3)=0$
$x=0$ (voldoet) $\vee\ x=3$ (voldoet).

5c $\frac{2x-1}{x-1}=\frac{2x-1}{2x+1}$ ($x\neq1$ en $2x\neq-1 \Rightarrow x\neq-\frac{1}{2}$)
$(2x-1)(2x+1)=(2x-1)(x-1)$
$2x-1=0 \vee 2x+1=x-1$
$2x=1 \vee x=-2$
$x=\frac{1}{2}$ (voldoet) $\vee\ x=-2$ (voldoet).

5d $\frac{2x-1}{x-1}=\frac{x+1}{x-1}\ (x\neq 1)\Rightarrow 2x-1=x+1\Rightarrow x=2$ (voldoet).

6 Neem $C=0$ en $D\neq 0$ in $\frac{A}{B}=\frac{C}{D}$ ($B\neq 0$ en $D\neq 0$), dan krijg je $\frac{A}{B}=0\ (B\neq 0)\Rightarrow A=0\ (B\neq 0)$.
Neem $C=A$ in $\frac{A}{B}=\frac{C}{D}$ ($B\neq 0$ en $D\neq 0$), dan krijg je $\frac{A}{B}=\frac{A}{D}$ ($B\neq 0$ en $D\neq 0$) $\Rightarrow A=0\vee B=D$ ($B\neq 0$ en $D\neq 0$).
Neem $D=B$ in $\frac{A}{B}=\frac{C}{D}$ ($B\neq 0$ en $D\neq 0$), dan krijg je $\frac{A}{B}=\frac{C}{B}\ (B\neq 0)\Rightarrow A=C\ (B\neq 0)$.

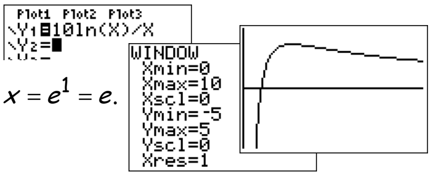
7a $\frac{x^2+3}{2x}=x-1\ (x\neq 0)$
$x^2+3=2x(x-1)$
$x^2+3=2x^2-2x$
$-x^2+2x+3=0$
$x^2-2x-3=0$
$(x-3)(x+1)=0$
$x=3$ (voldoet) $\vee$ $x=-1$ (voldoet).

7c $\frac{\cos(2x)}{4x}=\frac{\cos(2x)}{2x+\pi}\ (x\neq 0 \text{ en } x\neq -\frac{1}{2}\pi)$
$\cos(2x)=0\vee 4x=2x+\pi$
$2x=\frac{1}{2}\pi+k\cdot\pi\vee 2x=\pi$
$x=\frac{1}{4}\pi+k\cdot\frac{1}{2}\pi$ (voldoen) $\vee$ $x=\frac{1}{2}\pi$ (voldoet).

7b $\frac{4^x-2^x-6}{2^x-4}=0\ (2^x\neq 4\Rightarrow x\neq {}^2\log(4)={}^2\log(2^2)=2)$
$4^x-2^x-6=0$
$(2^x)^2-1\cdot 2^x-6=0$
$(2^x-3)(2^x+2)=0$
$2^x=3\vee 2^x=-2$ (kan niet)
$x={}^2\log(3)$ (voldoet).

7d $\frac{\ln^2(x)}{1+\ln(x)}=\frac{2+\ln(x)}{1+\ln(x)}\ (\ln(x)\neq -1\Rightarrow x\neq e^{-1}=\frac{1}{e})$
$\ln^2(x)=2+\ln(x)$
$\ln^2(x)-1\cdot\ln(x)-2=0$
$(\ln(x)-2)(\ln(x)+1)=0$
$\ln(x)=2\vee\ln(x)=-1$
$x=e^2$ (voldoet) $\vee$ $x=e^{-1}=\frac{1}{e}$ (voldoet niet).

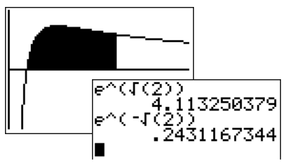
8a $f(x)=\frac{10\ln(x)}{x}\ (x>0)\Rightarrow f'(x)=\frac{x\cdot\frac{10}{x}-10\ln(x)\cdot 1}{x^2}=\frac{10-10\ln(x)}{x^2}$.
$f'(x)=0\Rightarrow\frac{10-10\ln(x)}{x^2}=0$ (teller $=0$) $\Rightarrow 10-10\ln(x)=0\Rightarrow\ln(x)=1\Rightarrow x=e^1=e$.
$f(e)=\frac{10\ln(e)}{e}=\frac{10}{e}$. Dus de top is $(e,\ \frac{10}{e})$.



8b $f'(x)=\frac{10-10\ln(x)}{x^2}\ (x>0)\Rightarrow f''(x)=\frac{x^2\cdot-\frac{10}{x}-(10-10\ln(x))\cdot 2x}{x^4}=\frac{-10x-20x+20x\ln(x)}{x^4}=\frac{-30x+20x\ln(x)}{x^4}=\frac{-30+20\ln(x)}{x^3}$.
$f''(x)=0\Rightarrow\frac{-30+20\ln(x)}{x^3}=0$ (teller $=0$) $\Rightarrow 20\ln(x)-30=0\Rightarrow 20\ln(x)=30\Rightarrow\ln(x)=1\frac{1}{2}\Rightarrow x=e^{1\frac{1}{2}}=e\sqrt{e}$.
$f(e\sqrt{e})=\frac{10\ln(e^{1\frac{1}{2}})}{e\sqrt{e}}=\frac{10\cdot 1\frac{1}{2}}{e\sqrt{e}}=\frac{15}{e\sqrt{e}}$. Dus het buigpunt is $(e\sqrt{e},\ \frac{15}{e\sqrt{e}})$.

8c Raaklijn door $O(0,0)\Rightarrow$ de x-coördinaat van het raakpunt volgt uit $f'(x)=\frac{f(x)}{x}$.
$\frac{10-10\ln(x)}{x^2}=\frac{10\ln(x)}{x^2}\Rightarrow 10-10\ln(x)=10\ln(x)\Rightarrow -20\ln(x)=-10\Rightarrow\ln(x)=\frac{1}{2}\Rightarrow x=e^{\frac{1}{2}}=\sqrt{e}$.
$\text{rc}_k=f'(\sqrt{e})=\frac{10-10\cdot\frac{1}{2}}{e}=\frac{5}{e}$. Dus k: $y=\frac{5}{e}x$.

8d $F(x)=5\ln^2(x)\Rightarrow F'(x)=5\cdot 2\ln(x)\cdot\frac{1}{x}=\frac{10\ln(x)}{x}=f(x)$. Dus F is een primitieve van f.
$O(V)=\int_1^a f(x)\,dx=\left[F(x)\right]_1^a=\left[5\ln^2(x)\right]_1^a=5\ln^2(a)-5\ln^2(1)=5\ln^2(a)-0=5\ln^2(a)$.
$O(V)=10\Rightarrow 5\ln^2(a)=10\Rightarrow\ln^2(a)=2\Rightarrow\ln(a)=\sqrt{2}\vee\ln(a)=-\sqrt{2}\Rightarrow a=e^{\sqrt{2}}\vee a=e^{-\sqrt{2}}$ ($\approx 0{,}24$ voldoet niet).



9a $(2x+1)^2=(2x)^2+2\cdot 2x\cdot 1+1^2=4x^2+4x+1$.
9b $(3x-2)^2=(3x)^2+2\cdot 3x\cdot -2+(-2)^2=9x^2-12x+4$.
9c $(4x+3)(4x-3)=(4x)^2-12x+12x-3^2=16x^2-9$.
9d $(x+2)^3=(x+2)\cdot(x+2)^2=(x+2)\cdot(x^2+4x+4)=x^3+4x^2+4x+2x^2+8x+8=x^3+6x^2+12x+8$.

10a $(3x\sqrt{2}-\sqrt{5})^2=18x^2-6x\sqrt{10}+5$.
10b $(2x-1)^3=(2x-1)\cdot(2x-1)^2=(2x-1)\cdot(4x^2-4x+1)=8x^3-8x^2+2x-4x^2+4x-1=8x^3-12x^2+6x-1$.
10c $\frac{2x^5-32x}{x^2-4}\ (x^2\neq 4\Rightarrow x\neq 2\wedge x\neq -2)=\frac{2x(x^4-16)}{x^2-4}=\frac{2x(x^2+4)(x^2-4)}{x^2-4}=2x(x^2+4)\ (x\neq 2\wedge x\neq -2)$.

10d $(4x\sqrt{2}-3)(4x\sqrt{2}+3)=32x^2-9.$

10e $(2^x+1)^3=(2^x+1)\cdot(2^x+1)^2=(2^x+1)\cdot(2^{2x}+2\cdot 2^x+1)=2^{3x}+2\cdot 2^{2x}+2^x+2^{2x}+2\cdot 2^x+1=2^{3x}+3\cdot 2^{2x}+3\cdot 2^x+1.$

10f $\frac{x^4+4x^2+4}{x^4-4}$ $(x^4\neq 4\Rightarrow x^2\neq 2\wedge x^2\neq -2\Rightarrow x\neq\sqrt{2}\wedge x\neq -\sqrt{2})=\frac{(x^2+2)^2}{(x^2+2)(x^2-2)}=\frac{(x^2+2)}{(x^2-2)}$ $(x\neq\sqrt{2}\wedge x\neq -\sqrt{2}).$

11a k: $y=mx+n$ door (a,a^2) en (b,b^2) $\text{rc}_k=m=\frac{b^2-a^2}{b-a}=\frac{(b+a)(b-a)}{b-a}=b+a=a+b.$
k: $y=(a+b)x+n$ door $(a,a^2)\Rightarrow a^2=(a+b)\cdot a+n\Rightarrow a^2=a^2+ab+n\Rightarrow -ab=n.$ Dus k: $y=(a+b)x-ab.$

11b $O(V)=\int_a^b((a+b)x-ab-x^2)\,dx=\left[(a+b)\cdot\frac{1}{2}x^2-abx-\frac{1}{3}x^3\right]_a^b$
$=\frac{1}{2}(a+b)b^2-ab\cdot b-\frac{1}{3}b^3-(\frac{1}{2}(a+b)a^2-ab\cdot a-\frac{1}{3}a^3)=\frac{1}{2}ab^2+\frac{1}{2}b^3-ab^2-\frac{1}{3}b^3-\frac{1}{2}a^3-\frac{1}{2}a^2b+a^2b+\frac{1}{3}a^3$
$=-\frac{1}{6}a^3+\frac{1}{2}a^2b-\frac{1}{2}ab^2+\frac{1}{6}b^3$ ❶
$\frac{1}{6}(b-a)^3=\frac{1}{6}(b-a)\cdot(b-a)^2=\frac{1}{6}(b-a)\cdot(b^2-2ab+a^2)=\frac{1}{6}(b^3-2ab^2+a^2b-ab^2+2a^2b-a^3)$
$=\frac{1}{6}(b^3-3ab^2+3a^2b-a^3)=\frac{1}{6}b^3-\frac{1}{2}ab^2+\frac{1}{2}a^2b-\frac{1}{6}a^3$ ❷. Uit ❶ en ❷ $\Rightarrow O(V)=\frac{1}{6}(b-a)^3.$

12 k: $y=mx+n$ door $(-2,0)$ en $(a,4-a^2)\Rightarrow \text{rc}_k=m=\frac{4-a^2-0}{a--2}=\frac{4-a^2}{a+2}=\frac{(2+a)(2-a)}{a+2}=2-a.$
k: $y=(2-a)x+n$ door $(-2,0)\Rightarrow 0=(2-a)\cdot -2+n\Rightarrow 2(2-a)=n.$ Dus k: $y=(2-a)x+4-2a.$
$O(V)=\int_{-2}^{a}(4-x^2-((2-a)x+4-2a))\,dx=\int_{-2}^{a}(4-x^2-2x+ax-4+2a)\,dx=\int_{-2}^{a}(-x^2-2x+ax+2a)\,dx$
$=\left[-\frac{1}{3}x^3-x^2+\frac{1}{2}ax^2+2ax\right]_{-2}^{a}=-\frac{1}{3}a^3-a^2+\frac{1}{2}a^3+2a^2-(-\frac{1}{3}\cdot(-2)^3-(-2)^2+\frac{1}{2}a\cdot(-2)^2+2a\cdot(-2))$
$=-\frac{1}{3}a^3-a^2+\frac{1}{2}a^3+2a^2-(\frac{8}{3}-4+2a-4a)=\frac{1}{6}a^3+a^2-(\frac{8}{3}-4-2a)=\frac{1}{6}a^3+a^2+2a+\frac{4}{3}$ ❶
$\frac{1}{6}(a+2)^3=\frac{1}{6}(a+2)\cdot(a+2)^2=\frac{1}{6}(a+2)\cdot(a^2+4a+4)=\frac{1}{6}(a^3+4a^2+4a+2a^2+8a+8)$
$=\frac{1}{6}(a^3+6a^2+12a+8)=\frac{1}{6}a^3+a^2+2a+\frac{4}{3}$ ❷. Uit ❶ en ❷ $\Rightarrow O(V)=\frac{1}{6}(a+2)^3.$

13a $y=2x-\frac{1}{x}$ $(x\neq 0)=2x\cdot\frac{x}{x}-\frac{1}{x}=\frac{2x^2}{x}-\frac{1}{x}=\frac{2x^2-1}{x}.$

13b $y=\frac{x}{x+1}+\frac{x}{x+2}$ $(x\neq -1\wedge x\neq -2)=\frac{x(x+2)}{(x+1)(x+2)}+\frac{x(x+1)}{(x+1)(x+2)}=\frac{x^2+2x}{(x+1)(x+2)}+\frac{x^2+x}{(x+1)(x+2)}=\frac{2x^2+3x}{(x+1)(x+2)}.$

13c $y=\frac{x}{\left(\frac{2}{x}\right)}$ $(x\neq 0)=x\cdot\frac{x}{2}=\frac{x^2}{2}.$

13d $y=(x+1)\cdot\frac{x+2}{x+3}$ $(x\neq -3)=\frac{(x+1)(x+2)}{x+3}=\frac{x^2+3x+2}{x+3}.$

13e $y=\frac{2}{x}\cdot\frac{x+2}{x+3}$ $(x\neq 0\wedge x\neq -3)=\frac{2(x+2)}{x(x+3)}=\frac{2x+4}{x(x+3)}.$

13f $y=\frac{x+1}{\left(\frac{x+2}{x-1}\right)}$ $(x\neq -2\wedge x\neq 1)=(x+1)\cdot\frac{x-1}{x+2}=\frac{(x+1)(x-1)}{x+2}=\frac{x^2-1}{x+2}.$

14 $y=\frac{20}{x-1}\cdot\left(4-\frac{2}{x-1}\right)$ $(x\neq 1)=\frac{20}{x-1}\cdot\left(\frac{4(x-1)}{x-1}-\frac{2}{x-1}\right)=\frac{20}{x-1}\cdot\frac{4x-4-2}{x-1}=\frac{20(4x-6)}{(x-1)^2}=\frac{80x-120}{(x-1)^2}.$

15a $y=\frac{20}{x}-\frac{5}{2x}$ $(x\neq 0)=\frac{40}{2x}-\frac{5}{2x}=\frac{35}{2x}.$

15b $y=\frac{10}{x-1}-x^2$ $(x\neq 1)=\frac{10}{x-1}-\frac{x^2(x-1)}{x-1}=\frac{10-x^2(x-1)}{x-1}=\frac{10-x^3+x^2}{x-1}=\frac{-x^3+x^2+10}{x-1}.$

15c $y=\frac{2x^2}{\left(\frac{x+1}{x-1}\right)}$ $(x\neq -1\wedge x\neq 1)=2x^2\cdot\frac{x-1}{x+1}=\frac{2x^2(x-1)}{x+1}=\frac{2x^3-2x^2}{x+1}.$

15d $y=\frac{x}{x-1}\cdot\left(x+\frac{1}{x-1}\right)$ $(x\neq 1)=\frac{x}{x-1}\cdot\left(\frac{x(x-1)}{x-1}+\frac{1}{x-1}\right)=\frac{x}{x-1}\cdot\frac{x^2-x+1}{x-1}=\frac{x^3-x^2+x}{(x-1)^2}.$

15e $y=\frac{5}{x-2}\cdot\frac{6}{x+2}$ $(x\neq 2\wedge x\neq -2)=\frac{30}{(x-2)(x+2)}.$

15f $y=\frac{\left(\frac{x+1}{2x}\right)}{x-1}$ $(x\neq 0\wedge x\neq 1)=\frac{x+1}{2x}\cdot\frac{1}{x-1}=\frac{x+1}{2x(x-1)}.$

16a $y = \frac{\ln(x)}{x} - \frac{2\ln(x)}{3x}\ (x>0) = \frac{3\ln(x)}{3x} - \frac{2\ln(x)}{3x} = \frac{\ln(x)}{3x}.$

16b $y = \frac{e^x}{x-1} - 2e^x\ (x \neq 1) = \frac{e^x}{x-1} - \frac{2e^x(x-1)}{x-1} = \frac{e^x - 2e^x(x-1)}{x-1} = \frac{e^x - 2xe^x + 2e^x}{x-1} = \frac{-2xe^x + 3e^x}{x-1} = \frac{(-2x+3)e^x}{x-1}.$

16c $y = \frac{e^x - 2}{\left(\frac{e^x+1}{e^x+2}\right)} = (e^x - 2)\cdot\frac{e^x+2}{e^x+1} = \frac{(e^x-2)(e^x+2)}{e^x+1} = \frac{e^{2x}-4}{e^x+1}.$

16d $y = \frac{2^x}{2^x+4}\left(2^x + \frac{1}{2^x}\right) = \frac{2^x \cdot 2^x}{2^x+4} + \frac{2^x}{2^x+4}\cdot\frac{1}{2^x} = \frac{2^{2x}}{2^x+4} + \frac{1}{2^x+4} = \frac{2^{2x}+1}{2^x+4}.$

16e $y = \frac{e^x}{e^x-1}\cdot\frac{e^{2x}}{e^x+1}\ (e^x \neq 1 \Rightarrow x \neq 0) = \frac{e^x \cdot e^{2x}}{(e^x-1)(e^x+1)} = \frac{e^{3x}}{(e^x-1)(e^x+1)}.$

16f $y = \frac{\left(\frac{5+\ln(x)}{x}\right)}{x}\ (x>0) = \frac{5+\ln(x)}{x}\cdot\frac{1}{x} = \frac{5+\ln(x)}{x^2}.$

17a
$\frac{4e^x}{e^x+1} = \frac{e^x}{e^x-1}\ (e^x \neq 1 \Rightarrow x \neq 0)$
$4e^x(e^x-1) = e^x(e^x+1)$
$4(e^x-1) = e^x+1$
$4e^x - 4 = e^x + 1$
$3e^x = 5$
$e^x = \frac{5}{3}$
$x = \ln(\frac{5}{3}).$

17b
$\frac{4e^x}{e^x+1}\cdot\frac{e^x}{e^x-1} = 6\ (x \neq 0)$
$\frac{4e^{2x}}{e^{2x}-1} = 6$
$4e^{2x} = 6e^{2x} - 6$
$-2e^{2x} = -6$
$e^{2x} = 3$
$2x = \ln(3)$
$x = \frac{1}{2}\ln(3).$

17c
$\frac{4e^x}{e^x+1} - \frac{e^x}{e^x-1} = 3\ (x \neq 0)$
(vermenigvuldigen met $(e^x+1)(e^x-1)$)
$4e^x(e^x-1) - e^x(e^x+1) = 3(e^x+1)(e^x-1)$
$4e^{2x} - 4e^x - e^{2x} - e^x = 3(e^{2x}-1)$
$3e^{2x} - 5e^x = 3e^{2x} - 3$
$-5e^x = -3$
$e^x = \frac{3}{5}$
$x = \ln(\frac{3}{5}).$

17d
$f'(x) + g'(x) = 0\ (e^x \neq 1 \Rightarrow x \neq 0)$
$\frac{(e^x+1)\cdot 4e^x - 4e^x \cdot e^x}{(e^x+1)^2} + \frac{(e^x-1)\cdot e^x - e^x \cdot e^x}{(e^x-1)^2} = 0$
$\frac{4e^{2x} + 4e^x - 4e^{2x}}{(e^x+1)^2} + \frac{e^{2x} - e^x - e^{2x}}{(e^x-1)^2} = 0$
$\frac{4e^x}{(e^x+1)^2} - \frac{e^x}{(e^x-1)^2} = 0$
$\frac{4e^x}{(e^x+1)^2} = \frac{e^x}{(e^x-1)^2}$ (e^x is alleen positief)
$\frac{4}{(e^x+1)^2} = \frac{1}{(e^x-1)^2}$
$4(e^x-1)^2 = (e^x+1)^2$
$2(e^x-1) = e^x+1 \vee 2(e^x-1) = -(e^x+1)$
$2e^x - 2 = e^x + 1 \vee 2e^x - 2 = -e^x - 1$
$e^x = 3 \vee 3e^x = 1$
$e^x = 3 \vee e^x = \frac{1}{3}$
$x = \ln(3) \vee x = \ln(\frac{1}{3}).$

18 $y = \frac{1+\frac{3}{x}}{x+1}\ (x \neq 0 \wedge x \neq -1) = \frac{1+\frac{3}{x}}{x+1}\cdot\frac{x}{x} = \frac{x+3}{x(x+1)} \Rightarrow$ I is correct.
II is niet correct, want de factor $(x+1)$ moet in de noemer staan (zie 18I).
$y = \frac{1+\frac{3}{x}}{x+1}\ (x \neq 0 \wedge x \neq -1) = \frac{x+3}{x(x+1)}$ (zie 18I) $= \frac{x}{x(x+1)} + \frac{3}{x(x+1)} = \frac{1}{x+1} + \frac{3}{x(x+1)} \Rightarrow$ III is correct.
IV is niet correct, want de factor $(x+1)$ moet in de noemer van de tweede breuk staan (zie 18III).

19a $y = \frac{x + \frac{3}{x+1}}{x}\ (x \neq 0 \wedge x \neq -1) = \frac{x + \frac{3}{x+1}}{x}\cdot\frac{x+1}{x+1} = \frac{x(x+1)+3}{x(x+1)} = \frac{x^2+x+3}{x(x+1)}.$

19b $y = \frac{10x}{p + \frac{x^2}{2p}}\ (p \neq 0) = \frac{10x}{p+\frac{x^2}{2p}}\cdot\frac{2p}{2p} = \frac{20px}{2p^2+x^2}.$

19c $y = \frac{10 + \frac{5}{x-1}}{6 - \frac{3}{x-1}}\ (x \neq 1 \wedge \frac{3}{x-1} \neq 6 \Rightarrow x - 1 \neq \frac{1}{2} \Rightarrow x \neq 1\frac{1}{2}) = \frac{10+\frac{5}{x-1}}{6-\frac{3}{x-1}}\cdot\frac{x-1}{x-1} = \frac{10(x-1)+5}{6(x-1)-3} = \frac{10x-10+5}{6x-6-3} = \frac{10x-5}{6x-9}.$

19d $y = \frac{\frac{3x}{x+4} - 5}{\frac{2x}{x+4} - x + 5}\ (x \neq -4) = \frac{\frac{3x}{x+4}-5}{\frac{2x}{x+4}-x+5}\cdot\frac{x+4}{x+4} = \frac{3x-5(x+4)}{2x+(-x+5)(x+4)} = \frac{3x-5x-20}{2x-x^2-4x+5x+20} = \frac{-2x-20}{-x^2+3x+20} = \frac{2x+20}{x^2-3x-20}.$

20a $N = \frac{600a}{3b - \frac{a^2}{4b}}\ (b \neq 0 \wedge 3b \neq \frac{a^2}{4b} \Rightarrow 12b^2 \neq a^2) = \frac{600a}{3b-\frac{a^2}{4b}}\cdot\frac{4b}{4b} = \frac{2400ab}{12b^2 - a^2}.$

20b $A = 25x + 20\cdot\frac{\frac{50}{x^2+1}}{x}\ (x \neq 0) = 25x + 20\cdot\frac{\frac{50}{x^2+1}}{x}\cdot\frac{x^2+1}{x^2+1} = 25x + 20\cdot\frac{50}{x(x^2+1)} = 25x + \frac{1000}{x(x^2+1)}.$

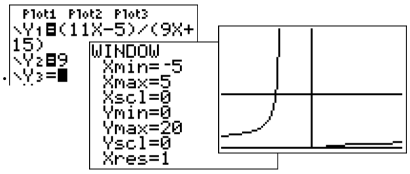
20c $K = \left(50 + \frac{150}{\frac{p}{q}+5}\right)\cdot p\ (q \neq 0 \wedge \frac{p}{q}+5 \neq 0) = 50p\cdot\frac{p+5q}{p+5q} + \frac{150p}{\frac{p}{q}+5}\cdot\frac{q}{q} = \frac{50p^2+250pq}{p+5q} + \frac{150pq}{p+5q} = \frac{50p^2+400pq}{p+5q}.$

21a $p = \frac{3x}{x+5}$ $(x \neq -5)$ invullen in $N = \frac{4p-1}{2p+3}$ $(2p+3 \neq 0)$ geeft $N = \frac{4 \cdot \frac{3x}{x+5} - 1}{2 \cdot \frac{3x}{x+5} + 3} \cdot \frac{x+5}{x+5} = \frac{12x-(x+5)}{6x+3(x+5)} = \frac{11x-5}{9x+15}$ $(x \neq -\frac{15}{9} = -1\frac{2}{3})$.

21b $N > 9 \Rightarrow \frac{11x-5}{9x+15} > 9$ $(x \neq -\frac{15}{9})$.

$\frac{11x-5}{9x+15} = 9 \Rightarrow 11x - 5 = 9(9x+15) \Rightarrow 11x - 5 = 81x + 135 \Rightarrow -70x = 140 \Rightarrow x = -2.$

$N > 9$ (zie een plot) $\Rightarrow -2 < x < -1\frac{2}{3}$.



22a $N = \frac{x^2+5x-6}{x}$ $(x \neq 0) = \frac{x^2}{x} + \frac{5x}{x} - \frac{6}{x} = x + 5 - \frac{6}{x}$.

22b $A = \frac{5x^2+1000}{x}$ $(x \neq 0) = \frac{5x^2}{x} + \frac{1000}{x} = 5x + \frac{1000}{x}$.

22c $K = \frac{6t^2+12t+1500}{3t}$ $(t \neq 0) = \frac{6t^2}{3t} + \frac{12t}{3t} + \frac{1500}{3t} = 2t + 4 + \frac{500}{t}$.

22d $F = \frac{5a^2+8a}{2a^2}$ $(a \neq 0) = \frac{5a^2}{2a^2} + \frac{8a}{2a^2} = 2\frac{1}{2} + \frac{4}{a}$.

22e $N = \frac{6p^2-3p-1}{2p}$ $(p \neq 0) = \frac{6p^2}{2p} - \frac{3p}{2p} - \frac{1}{2p} = 3p - 1\frac{1}{2} - \frac{1}{2p}$.

23 $y = \frac{2}{x} \Rightarrow xy = 2 \Rightarrow x = \frac{2}{y}$.

24a
$A = \frac{B}{B+2}$
$A(B+2) = B$
$AB + 2A = B$
$AB - B = -2A$
$B(A-1) = -2A$
$B = -\frac{2A}{A-1}$.

24b
$P = \frac{Q-5}{Q}$
$PQ = Q - 5$
$PQ - Q = -5$
$Q(P-1) = -5$
$Q = -\frac{5}{P-1}$.

24c
$R = \frac{F-2}{F-1}$
$R(F-1) = F - 2$
$RF - R = F - 2$
$RF - F = R - 2$
$F(R-1) = R - 2$
$F = \frac{R-2}{R-1}$.

24d
$L = 320 - \frac{18}{q-1}$
$\frac{18}{q-1} = \frac{320-L}{1}$
$\frac{18}{320-L} = \frac{q-1}{1}$
$q - 1 = \frac{18}{320-L}$
$q = 1 + \frac{18}{320-L}$.

25a
$\frac{1}{a} = 2 + \frac{1}{b}$
$\frac{1}{a} = \frac{2b}{b} + \frac{1}{b}$
$\frac{1}{a} = \frac{2b+1}{b}$
(links en rechts het omgekeerde nemen)
$\frac{a}{1} = a = \frac{b}{2b+1}$.

25b
$\frac{1}{a} = 2 + \frac{1}{b}$
$\frac{1}{a} - 2 = \frac{1}{b}$
$\frac{1}{a} - \frac{2a}{a} = \frac{1}{b}$
$\frac{1-2a}{a} = \frac{1}{b}$
$\frac{a}{1-2a} = \frac{b}{1} = b$.

26a
$\frac{1}{p} = 5 - \frac{2}{q}$
$\frac{1}{p} = \frac{5q}{q} - \frac{2}{q}$
$\frac{1}{p} = \frac{5q-2}{q}$
$p = \frac{q}{5q-2}$.

$\frac{1}{p} = 5 - \frac{2}{q}$
$\frac{2}{q} = \frac{5p}{p} - \frac{1}{p}$
$\frac{2}{q} = \frac{5p-1}{p}$ (links en rechts :2)
$\frac{1}{q} = \frac{5p-1}{2p}$
$q = \frac{2p}{5p-1}$.

26b
$\frac{1}{m} = \frac{1}{2} - \frac{3}{n}$
$\frac{1}{m} = \frac{n}{2n} - \frac{6}{2n}$
$\frac{1}{m} = \frac{n-6}{2n}$
$m = \frac{2n}{n-6}$.

$\frac{1}{m} = \frac{1}{2} - \frac{3}{n}$
$\frac{3}{n} = \frac{1}{2} - \frac{1}{m}$
$\frac{3}{n} = \frac{m}{2m} - \frac{2}{2m}$
$\frac{3}{n} = \frac{m-2}{2m}$ (:3)
$\frac{1}{n} = \frac{m-2}{6m}$
$n = \frac{6m}{m-2}$.

27a
$\frac{t-2}{t-3} \cdot P = \frac{t}{t-1}$
$P = \frac{t}{t-1} \cdot \frac{t-3}{t-2}$
$P = \frac{t(t-3)}{(t-1)(t-2)}$.

27b
$\frac{3x}{x+y} = 5 - y$
$3x = (5-y) \cdot (x+y)$
$3x = 5x + 5y - xy - y^2$
$xy - 2x = 5y - y^2$
$x(y-2) = 5y - y^2$
$x = \frac{-y^2+5y}{y-2}$.

27c
$\frac{1500-N}{N} = 50 \cdot 0{,}95^t$
$1500 - N = 50 \cdot 0{,}95^t \cdot N$
$1500 = 50 \cdot 0{,}95^t \cdot N + N$
$1500 = N(50 \cdot 0{,}95^t + 1)$
$N = \frac{1500}{50 \cdot 0{,}95^t + 1}$.

27d
$K = 90 - \frac{2N}{N+0{,}2}$
$\frac{2N}{N+0{,}2} = 90 - K$
$2N = (90-K)(N+0{,}2)$
$2N = 90N + 18 - KN - 0{,}2K$
$2N + KN - 90N = 18 - 0{,}2K$
$N(K-88) = 18 - 0{,}2K$
$N = \frac{18-0{,}2K}{K-88}$.

28a $F = \frac{1}{K} + \frac{1}{2K} = \frac{2}{2K} + \frac{1}{2K} = \frac{3}{2K} \Rightarrow \frac{F}{3} = \frac{1}{2K} \Rightarrow 2K = \frac{3}{F} \Rightarrow K = \frac{3}{2F}$.

28b $\frac{1}{T} = 10 - \frac{2}{S} = \frac{10S}{S} - \frac{2}{S} = \frac{10S-2}{S} \Rightarrow T = \frac{S}{10S-2}$.

28c $\frac{1}{N} + 3 = \frac{2R+2}{5R+2} \Rightarrow \frac{1}{N} = \frac{2R+2}{5R+2} - 3 = \frac{2R+2}{5R+2} - \frac{3(5R+2)}{5R+2} = \frac{2R+2-15R-6}{5R+2} = \frac{-13R-4}{5R+2} \Rightarrow N = \frac{5R+2}{-13R-4}$.

29a $\left.\begin{array}{l} K = \frac{500}{A} + 40A + \frac{60}{B} + 25B \\ AB = 30 \Rightarrow B = \frac{30}{A} \end{array}\right\} \Rightarrow K = \frac{500}{A} + 40A + 60 \cdot \frac{A}{30} + 25 \cdot \frac{30}{A} = \frac{500}{A} + 40A + 2A + \frac{750}{A} = \frac{1250}{A} + 42A.$

29b $\left.\begin{array}{l} F = \frac{80}{A-1} + 10A + \frac{40}{AB} + 5A^3B \\ A^2B = 20 \Rightarrow B = \frac{20}{A^2} \end{array}\right\} \Rightarrow F = \frac{80}{A-1} + 10A + \frac{40}{A} \cdot \frac{A^2}{20} + 5A^3 \cdot \frac{20}{A^2} = \frac{80}{A-1} + 10A + 2A + 100A = \frac{80}{A-1} + 112A.$

29c $\left.\begin{array}{l} N = 2PQ + P\left(PQ - \frac{2}{P}\right) \\ P^2Q = 10 \Rightarrow PQ = \frac{10}{P} \end{array}\right\} \Rightarrow N = 2 \cdot \frac{10}{P} + P\left(\frac{10}{P} - \frac{2}{P}\right) = \frac{20}{P} + P \cdot \frac{8}{P} = \frac{20}{P} + 8 \Rightarrow \frac{20}{P} = N - 8 \Rightarrow \frac{P}{20} = \frac{1}{N-8} \Rightarrow P = \frac{20}{N-8}.$

30a $y = \frac{1}{\sqrt{x}} + \sqrt{x} = \frac{1}{\sqrt{x}} + \frac{\sqrt{x}}{1} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{1}{\sqrt{x}} + \frac{x}{\sqrt{x}} = \frac{1+x}{\sqrt{x}} = \frac{x+1}{\sqrt{x}}.$

30b $K = 2\sqrt{p+2} + \sqrt{\frac{1}{9}p + \frac{2}{9}} = 2\sqrt{p+2} + \sqrt{\frac{1}{9}(p+2)} = 2\sqrt{p+2} + \frac{1}{3}\sqrt{p+2} = 2\frac{1}{3}\sqrt{p+2}.$

31a $y = \sqrt{25x} - \sqrt{x} = 5\sqrt{x} - \sqrt{x} = 4\sqrt{x}.$

31b $y = \sqrt{54x} - \sqrt{24x} = \sqrt{9 \cdot 6x} - \sqrt{4 \cdot 6x} = 3\sqrt{6x} - 2\sqrt{6x} = \sqrt{6x}.$

31c $N = \sqrt{8a} + \sqrt{\frac{1}{2}a} = \sqrt{4 \cdot 2a} + \sqrt{\frac{1}{2}a \cdot \frac{2}{2}} = 2\sqrt{2a} + \sqrt{\frac{1}{4} \cdot 2a} = 2\sqrt{2a} + \frac{1}{2}\sqrt{2a} = 2\frac{1}{2}\sqrt{2a}.$

31d $N = \sqrt{20a} + \sqrt{\frac{9a}{5}} = \sqrt{4 \cdot 5a} + \sqrt{\frac{9a}{5} \cdot \frac{5}{5}} = 2\sqrt{5a} + \sqrt{\frac{9}{5^2} \cdot 5a} = 2\sqrt{5a} + \frac{3}{5}\sqrt{5a} = 2\frac{3}{5}\sqrt{5a}.$

32a $y = (x+2)\sqrt{x^2+4} - 2\sqrt{x^2+4} = (x+2-2)\sqrt{x^2+4} = x\sqrt{x^2+4}.$

32b $y = \frac{3}{\sqrt{x}} + 2\sqrt{x} = \frac{3}{\sqrt{x}} + \frac{2\sqrt{x}}{1} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{3}{\sqrt{x}} + \frac{2x}{\sqrt{x}} = \frac{2x+3}{\sqrt{x}}.$

32c $N = \frac{2-t^2}{t} \cdot \sqrt{t} + t \cdot \sqrt{t} = (\frac{2-t^2}{t} + t) \cdot \sqrt{t} = (\frac{2-t^2}{t} + \frac{t^2}{t}) \cdot \sqrt{t} = \frac{2}{t} \cdot \sqrt{t}.$

32d $N = \frac{t+1}{\sqrt{2t+1}} - \sqrt{2t+1} = \frac{t+1}{\sqrt{2t+1}} - \frac{2t+1}{\sqrt{2t+1}} = \frac{-t}{\sqrt{2t+1}}.$

33a $A = \sqrt{12b} - 6\sqrt{\frac{3}{16}b} = \sqrt{4 \cdot 3b} - 6\sqrt{\frac{1}{16} \cdot 3b} = 2\sqrt{3b} - 6 \cdot \frac{1}{4}\sqrt{3b} = \frac{1}{2}\sqrt{3b}.$

33b $B = \frac{5a}{2\sqrt{a}} - \sqrt{2\frac{1}{4}a} = \frac{5a\sqrt{a}}{2a} - \sqrt{\frac{9}{4}a} = \frac{5}{2}\sqrt{a} - \frac{3}{2}\sqrt{a} = \sqrt{a}.$

33c $y = \frac{x^2+4}{\sqrt{x-2}} - x\sqrt{x-2} = \frac{x^2+4}{\sqrt{x-2}} - \frac{x(x-2)}{\sqrt{x-2}} = \frac{x^2+4}{\sqrt{x-2}} - \frac{x^2-2x}{\sqrt{x-2}} = \frac{2x+4}{\sqrt{x-2}}.$

33d $N = \frac{3t^2}{(t-1)\sqrt{t-1}} - 3\sqrt{t-1} = \frac{3t^2}{(t-1)\sqrt{t-1}} - \frac{3(t-1)^2}{(t-1)\sqrt{t-1}} = \frac{3t^2}{(t-1)\sqrt{t-1}} - \frac{3(t^2-2t+1)}{(t-1)\sqrt{t-1}} = \frac{3t^2}{(t-1)\sqrt{t-1}} - \frac{3t^2-6t+3}{(t-1)\sqrt{t-1}} = \frac{6t-3}{(t-1)\sqrt{t-1}}.$

34a $f(x) = x^2 \cdot \sqrt{2x+5} \Rightarrow f'(x) = 2x \cdot \sqrt{2x+5} + x^2 \cdot \frac{1}{2 \cdot \sqrt{2x+5}} \cdot 2 = \frac{2x(2x+5)}{\sqrt{2x+5}} + \frac{x^2}{\sqrt{2x+5}} = \frac{5x^2+10x}{\sqrt{2x+5}} \Rightarrow a = 5$ en $b = 10.$

34b $f'(x) = 0 \Rightarrow \frac{5x^2+10x}{\sqrt{2x+5}} = 0$ (teller = 0) $\Rightarrow 5x(x+2) = 0 \Rightarrow x = 0 \vee x = -2.$

$x_A = -2 \Rightarrow f(-2) = (-2)^2 \cdot \sqrt{2 \cdot -2 + 5} = 4 \cdot \sqrt{1} = 4.$ Dus $A(-2, 4).$

34c $F(x) = (px^2 + qx + r)(2x+5)^{1\frac{1}{2}} \Rightarrow F'(x) = (2px+q) \cdot (2x+5)^{1\frac{1}{2}} + (px^2+qx+r) \cdot 1\frac{1}{2}(2x+5)^{\frac{1}{2}} \cdot 2$

$= (2px+q) \cdot (2x+5) \cdot \sqrt{2x+5} + 3(px^2+qx+r) \cdot \sqrt{2x+5}$

$= (4px^2 + 10px + 2qx + 5q + 3px^2 + 3qx + 3r) \cdot \sqrt{2x+5}$

$= (7px^2 + (10p+5q)x + 5q + 3r) \cdot \sqrt{2x+5}.$

$F'(x) = f(x) \Rightarrow 7p = 1 \wedge 10p + 5q = 0 \wedge 5q + 3r = 0$

$p = \frac{1}{7} \wedge \frac{10}{7} + 5q = 0 \wedge 5q + 3r = 0$

$p = \frac{1}{7} \wedge 5q = -\frac{10}{7} \wedge -\frac{10}{7} + 3r = 0$

$p = \frac{1}{7} \wedge q = -\frac{2}{7} \wedge 3r = \frac{10}{7}$

$p = \frac{1}{7} \wedge q = -\frac{2}{7} \wedge r = \frac{10}{21}.$

34d $O(V) = \int_{-2\frac{1}{2}}^{0} f(x)\,dx = \left[(\frac{1}{7}x^2 - \frac{2}{7}x + \frac{10}{21})(2x+5)\sqrt{2x+5}\right]_{-2\frac{1}{2}}^{0} = \frac{10}{21} \cdot 5 \cdot \sqrt{5} - (\frac{1}{7} \cdot (-2\frac{1}{2})^2 - \frac{2}{7} \cdot -2\frac{1}{2} + \frac{10}{21}) \cdot 0 = \frac{50}{21}\sqrt{5}.$

35a $y = 2\sqrt{x}$ $(x \geq 0) \Rightarrow 2\sqrt{x} = y$ (kwadrateren) $\Rightarrow 4x = y^2$ $(y \geq 0) \Rightarrow x = \frac{1}{4}y^2$ $(y \geq 0)$.

35b $y = \sqrt{x-2}$ $(x - 2 \geq 0 \Rightarrow x \geq 2) \Rightarrow \sqrt{x-2} = y$ (kwadrateren) $\Rightarrow x - 2 = y^2$ $(y \geq 0) \Rightarrow x = y^2 + 2$ $(y \geq 0)$.

35c $y = 2\sqrt{x-2}$ $(x \geq 2) \Rightarrow 2\sqrt{x-2} = y$ (kwadrateren) $\Rightarrow 4(x-2) = y^2$ $(y \geq 0) \Rightarrow x - 2 = \frac{1}{4}y^2 \Rightarrow x = \frac{1}{4}y^2 + 2$ $(y \geq 0)$.

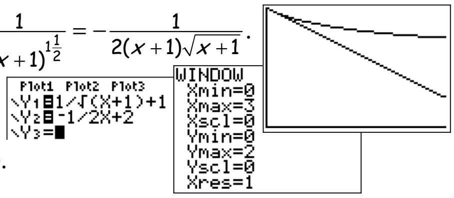
36a
$R = 2\sqrt{4A-1}$
$2\sqrt{4A-1} = R$ (kwadrateren)
$4(4A-1) = R^2$
$4A - 1 = \frac{1}{4}R^2 \Rightarrow 4A = \frac{1}{4}R^2 + 1 \Rightarrow A = \frac{1}{16}R^2 + \frac{1}{4}$.

36c
$x\sqrt{y} - 2x = 4$
$x\sqrt{y} = 2x + 4$
$\sqrt{y} = \frac{2x+4}{x}$ (kwadrateren)
$y = \frac{(2x+4)^2}{x^2}$.

36b
$A = 6 - \frac{2}{\sqrt{B}}$
$\frac{2}{\sqrt{B}} = 6 - A$
$\frac{2}{6-A} = \sqrt{B}$ (kwadrateren)
$B = \frac{4}{(6-A)^2}$.

36d
$\frac{p\sqrt{q}}{q} - \sqrt{p} = 4$
$\frac{p\sqrt{q}}{q} = 4 + \sqrt{p}$
$\frac{1}{\sqrt{q}} = \frac{4+\sqrt{p}}{p} \Rightarrow \frac{\sqrt{q}}{1} = \frac{p}{4+\sqrt{p}}$ (kwadrateren) $\Rightarrow q = \frac{p^2}{(4+\sqrt{p})^2}$.

37a $f(x) = \frac{1}{\sqrt{x+1}} + 1 = \frac{1}{(x+1)^{\frac{1}{2}}} + 1 = (x+1)^{-\frac{1}{2}} + 1 \Rightarrow f'(x) = -\frac{1}{2}(x+1)^{-1\frac{1}{2}} \cdot 1 = -\frac{1}{2(x+1)^{1\frac{1}{2}}} = -\frac{1}{2(x+1)\sqrt{x+1}}$.



$f(0) = \frac{1}{\sqrt{1}} + 1 = 1 + 1 = 2$ en $f'(0) = -\frac{1}{2 \cdot 1 \cdot \sqrt{1}} = -\frac{1}{2} \Rightarrow k$: $y = -\frac{1}{2}x + 2$.

$O(V) = \int_0^3 \left(\frac{1}{\sqrt{x+1}} + 1\right) \mathrm{d}x = \left[2\sqrt{x+1} + x\right]_0^3 = 2\sqrt{4} + 3 - (2\sqrt{1} + 0) = 4 + 3 - 2 = 5.$

De oppervlakte van het deel van V onder de lijn k is $\int_0^3 \left(-\frac{1}{2}x + 2\right) \mathrm{d}x = \left[-\frac{1}{4}x^2 + 2x\right]_0^3 = -\frac{9}{4} + 6 - (0+0) = \frac{15}{4}$.

De verhouding van de oppervlakten van beide vlakdelen is $\frac{15}{4} : \frac{5}{4} = 15 : 5 = 3 : 1$.

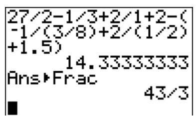
37b x vrijmaken in $y = \frac{1}{\sqrt{x+1}} + 1 \Rightarrow y - 1 = \frac{1}{\sqrt{x+1}} \Rightarrow \sqrt{x+1} = \frac{1}{y-1} \Rightarrow x + 1 = \frac{1}{(y-1)^2} \Rightarrow x = \frac{1}{(y-1)^2} - 1$.

$f(0) = \frac{1}{\sqrt{1}} + 1 = 1 + 1 = 2$ en $f(3) = \frac{1}{\sqrt{4}} + 1 = \frac{1}{2} + 1 = 1\frac{1}{2}$.

$$I(V) = I(\text{cilinder}) + \int_{1\frac{1}{2}}^{2} \pi x^2 \,\mathrm{d}y = \pi \cdot 3^2 \cdot \frac{3}{2} + \int_{1\frac{1}{2}}^{2} \pi \left(\frac{1}{(y-1)^2} - 1\right)^2 \mathrm{d}y = \frac{27}{2}\pi + \int_{1\frac{1}{2}}^{2} \pi \left(\frac{1}{(y-1)^4} - \frac{2}{(y-1)^2} + 1\right) \mathrm{d}y$$

$$= 13\frac{1}{2}\pi + \int_{1\frac{1}{2}}^{2} \pi \left((y-1)^{-4} - 2(y-1)^{-2} + 1\right) \mathrm{d}y = 13\frac{1}{2}\pi + \left[\pi\left(-\frac{1}{3}(y-1)^{-3} + 2(y-1)^{-1} + y\right)\right]_{1\frac{1}{2}}^{2}$$

$$= 13\frac{1}{2}\pi + \left[\pi\left(-\frac{1}{3(y-1)^3} + \frac{2}{y-1} + y\right)\right]_{1\frac{1}{2}}^{2} = 13\frac{1}{2}\pi + \pi\left(-\frac{1}{3} + \frac{2}{1} + 2\right) - \pi\left(-\frac{1}{\frac{3}{8}} + \frac{2}{\frac{1}{2}} + 1\frac{1}{2}\right)$$



$$= 13\frac{1}{2}\pi + 3\frac{2}{3}\pi - \pi\left(-\frac{8}{3} + 4 + 1\frac{1}{2}\right) = 13\frac{1}{2}\pi + 3\frac{2}{3}\pi - 2\frac{5}{6}\pi = 14\frac{1}{3}\pi.$$

38a $\frac{1}{x^3} = x^{-3}$.

38c $x\sqrt{x} = x^1 \cdot x^{\frac{1}{2}} = x^{1\frac{1}{2}}$.

38e $\frac{x^6}{x^2} = x^4$.

38b $\sqrt[4]{x^3} = x^{\frac{3}{4}}$.

38d $x^3 \cdot x^2 = x^5$.

38f $\frac{x}{\sqrt{x}} = x : \sqrt{x} = x^1 : x^{\frac{1}{2}} = x^{\frac{1}{2}}$.

39a
$x^5 = 18$
$x = \sqrt[5]{18}$.

39b
$\sqrt[3]{x} = 4$
$x = 4^3$
$x = 64$.

40a $y = \frac{6}{x} \cdot x^{1,4} = 6 \cdot \frac{x^{1,4}}{x} = 6x^{0,4}$.

40c $y = \left(2x^{0,4}\right)^3 \cdot 5 \cdot x^{0,5} = 8x^{1,2} \cdot 5x^{0,5} = 40x^{1,7}$.

40b $y = \frac{15}{x^2 \cdot \sqrt{x}} \cdot \sqrt[4]{x} = 15 \cdot \frac{x^{\frac{1}{4}}}{x^{2\frac{1}{2}}} = 15x^{-2\frac{1}{4}}$.

40d $y = \left(4\sqrt{x}\right)^3 \cdot \left(\frac{1}{2}x\right)^4 = \left(4x^{\frac{1}{2}}\right)^3 \cdot \frac{1}{16}x^4 = 64x^{1\frac{1}{2}} \cdot \frac{1}{16}x^4 = 4x^{5\frac{1}{2}}$.

41a $y = \frac{1}{3} \cdot \sqrt[3]{x} - 7$
$y + 7 = \frac{1}{3} \cdot \sqrt[3]{x}$
$3y + 21 = \sqrt[3]{x}$
$x = (3y + 21)^3.$

41b $P = \frac{1}{2} \cdot \sqrt[4]{2q-1} + 3$
$P - 3 = \frac{1}{2} \cdot \sqrt[4]{2q-1}$
$2P - 6 = \sqrt[4]{2q-1}$
$2q - 1 = (2P-6)^4$
$2q = (2P-6)^4 + 1$
$q = \frac{1}{2}(2P-6)^4 + \frac{1}{2}.$

41c $T = \frac{1}{16} \cdot S^4$
$16T = S^4$
$S = (16T)^{\frac{1}{4}}$
$S = 16^{\frac{1}{4}} \cdot T^{\frac{1}{4}}$
$S = \left(2^4\right)^{\frac{1}{4}} \cdot T^{\frac{1}{4}}$
$S = 2T^{\frac{1}{4}}.$

41d $A = 15 \cdot (4B)^{-1,6}$
$\frac{1}{15}A = (4B)^{-1,6}$
$4B = \left(\frac{1}{15}A\right)^{\frac{1}{-1,6}}$
$B = \frac{1}{4}\left(\frac{1}{15}A\right)^{\frac{1}{-1,6}}$
$B = \frac{1}{4} \cdot \left(\frac{1}{15}\right)^{\frac{1}{-1,6}} \cdot A^{\frac{1}{-1,6}}$
$B \approx 1,358 \cdot A^{-0,625}.$

```
1/4*(1/15)^(1/-1
.6)
         1.358304206
1/-1.6
                -.625
```

42a $P = 20 \cdot \left(3x^{0,6}\right)^2 \cdot \left(2y^3\right)^{0,8} = 20 \cdot 3^2 \cdot x^{1,2} \cdot 2^{0,8} \cdot y^{2,4} \approx 313,40x^{1,2}y^{2,4}.$

```
20*3²*2^0.8
         313.3982028
```

42b s elimineren (uitstoten) $\Rightarrow L = 0,6 \cdot (5t)^{0,35} \cdot \left(6 \cdot \frac{1}{3}t^3\right)^{0,18} = 0,6 \cdot 5^{0,35} \cdot t^{0,35} \cdot 2^{0,18} \cdot t^{0,54} \approx 1,19t^{0,89}.$

```
0.6*5^0.35*2^0.1
8
          1.19392254
3*0.18
                 .54
0.35+0.54
                 .89
```

42c t elimineren $\Rightarrow N = 0,12 \cdot \left(3 \cdot \frac{1}{2}k^{-0,4}\right)^{1,6} \cdot (5k)^{2,3} = 0,12 \cdot 1,5^{1,6} \cdot k^{-0,64} \cdot 5^{2,3} \cdot k^{2,3} \approx 9,30k^{1,66}.$

```
0.12*1.5^1.6*5^2
.3
         9.301613087
```

42d a elimineren $\Rightarrow y = \frac{1536}{\left(4 \cdot \sqrt{x^2+20}\right)^5} = 1536 \cdot \left(4 \cdot \left(x^2+20\right)^{0,5}\right)^{-5} = 1536 \cdot 4^{-5} \cdot \left(x^2+20\right)^{-2,5} = 1,5 \cdot \left(x^2+20\right)^{-2,5}.$

```
1536*4^-5
                 1.5
```

43a $V = 6 \cdot (5R)^{1,8}$
$\frac{1}{6}V = (5R)^{1,8}$
$\left(\frac{1}{6}V\right)^{\frac{1}{1,8}} = 5R$
$R = \frac{1}{5} \cdot \left(\frac{1}{6}V\right)^{\frac{1}{1,8}}$
$R = \frac{1}{5} \cdot \left(\frac{1}{6}\right)^{\frac{1}{1,8}} \cdot V^{\frac{1}{1,8}}$
$R \approx 0,07 \cdot V^{0,56}.$

```
1/5*(1/6)^(1/1.8
)
        .0739134962
1/1.8
        .5555555556
```

43b $S = \frac{2}{5} \cdot \sqrt[3]{4t} + 2$
$S - 2 = \frac{2}{5} \cdot \sqrt[3]{4t}$
$2,5 \cdot (S-2) = \sqrt[3]{4t}$
$(2,5 \cdot (S-2))^3 = 4t$
$t = \frac{1}{4} \cdot (2,5S - 5)^3.$

43c $O = 16a^2$
$\frac{1}{16}O = a^2$
$a = \left(\frac{1}{16}O\right)^{\frac{1}{2}} = \frac{1}{4}O^{\frac{1}{2}}$
$K = 8 \cdot \left(\frac{1}{4}O^{\frac{1}{2}}\right)^3$
$K = 8 \cdot \left(\frac{1}{4}\right)^3 \cdot O^{1\frac{1}{2}}$
$K = \frac{1}{8}O^{1\frac{1}{2}}.$

$K = 8a^3$
$\frac{1}{8}K = a^3$
$a = \left(\frac{1}{8}K\right)^{\frac{1}{3}} = \frac{1}{2}K^{\frac{1}{3}}$
$O = 16 \cdot \left(\frac{1}{2}K^{\frac{1}{3}}\right)^2$
$O = 16 \cdot \left(\frac{1}{2}\right)^2 \cdot K^{\frac{2}{3}}$
$O = 4 \cdot K^{\frac{2}{3}}.$

43d $L = 4 \cdot (r-3)^3$
$\frac{1}{4}L = (r-3)^3$
$r - 3 = \left(\frac{1}{4}L\right)^{\frac{1}{3}}$
$F = 2 \cdot \left(\left(\frac{1}{4}L\right)^{\frac{1}{3}}\right)^4$
$F = 2 \cdot \left(\frac{1}{4}L\right)^{1\frac{1}{3}}.$

$F = 2 \cdot (r-3)^4$
$\frac{1}{2}F = (r-3)^4$
$r - 3 = \left(\frac{1}{2}F\right)^{\frac{1}{4}}$
$L = 4 \cdot \left(\left(\frac{1}{2}F\right)^{\frac{1}{4}}\right)^3$
$L = 4 \cdot \left(\frac{1}{2}F\right)^{\frac{3}{4}}.$

44 $y = 2^x \cdot 2^{3x+1} = 2^x \cdot 2^{3x} \cdot 2^1 = 2^{4x} \cdot 2 = 2 \cdot \left(2^4\right)^x = 2 \cdot 16^x.$

45a $10^{\log(2)} = 10^{{}^{10}\log(2)} = 2$ ($10^{\cdots}$ en ${}^{10}\log\ldots$ heffen elkaar op) $e^{\ln(7)} = e^{{}^{e}\log(7)} = 7$ ($e^{\cdots}$ en $\ln\ldots$ heffen elkaars op).

45b $3 = 10^{\log(3)}$ ($10^{\cdots}$ en ${}^{10}\log\ldots$ zijn elkaars inverse) $5 = e^{\ln(5)}$ ($e^{\cdots}$ en $\ln\ldots$ zijn elkaars inverse).

46a $N = 25 \cdot 1,3^{4t-2} = 25 \cdot 1,3^{4t} \cdot 1,3^{-2} = 25 \cdot 1,3^{-2} \cdot \left(1,3^4\right)^t \approx 14,79 \cdot 2,86^t.$

```
25*1.3^-2
         14.79289941
1.3^4
              2.8561
```

46b $N = 180 \cdot 0,8^{5-t} = 180 \cdot 0,8^5 \cdot 0,8^{-t} = 180 \cdot 0,8^5 \cdot \left(0,8^{-1}\right)^t \approx 58,98 \cdot 1,25^t.$

```
180*0.8^5
             58.9824
0.8^-1
                1.25
```

47a $N = 3^t \cdot 3^{t+3} = 3^{2t+3} = 3^{2t} \cdot 3^3 = 3^3 \cdot \left(3^2\right)^t = 27 \cdot 9^t.$

47b $N = \left(\frac{1}{2}\right)^{t-3} \cdot \left(\frac{1}{2}\right)^{-3t+2} = \left(\frac{1}{2}\right)^{-2t-1} = \left(2^{-1}\right)^{-2t-1} = 2^{2t+1} = 2^{2t} \cdot 2^1 = 2 \cdot \left(2^2\right)^t = 2 \cdot 4^t.$

47c $N = 3^{2t-1}\cdot 9^{t+1} = 3^{2t-1}\cdot\left(3^2\right)^{t+1} = 3^{2t-1}\cdot 3^{2t+2} = 3^{4t+1} = 3^{4t}\cdot 3^1 = \left(3^4\right)^t\cdot 3 = 3\cdot 81^t.$

47d $N = \left(\frac{1}{4}\right)^{t+1}\cdot 2^{3t+4} = \left(2^{-2}\right)^{t+1}\cdot 2^{3t+4} = 2^{-2t-2}\cdot 2^{3t+4} = 2^{t+2} = 2^t\cdot 2^2 = 4\cdot 2^t.$

48a $y = 15\cdot 5^{x-1} = 15\cdot 5^x\cdot 5^{-1} = 15\cdot\frac{1}{5}\cdot\left(10^{\log(5)}\right)^x = 3\cdot 10^{x\cdot\log(5)} \approx 3\cdot 10^{0{,}70x}.$

```
log(5)
       .6989700043
```

48b $y = 15\cdot 5^{x-1} = 15\cdot 5^x\cdot 5^{-1} = 15\cdot\frac{1}{5}\cdot\left(e^{\ln(5)}\right)^x = 3\cdot e^{x\cdot\ln(5)} \approx 3\cdot e^{1{,}61x}.$

```
ln(5)
       1.609437912
```

```
37.2*1.7^-2
       12.87197232
3*log(1.7)
       .6913467641
```

48c $T = 37{,}2\cdot 1{,}7^{3t-2} = 37{,}2\cdot 1{,}7^{3t}\cdot 1{,}7^{-2} = 37{,}2\cdot 1{,}7^{-2}\cdot\left(10^{\log(1{,}7)}\right)^{3t} = 37{,}2\cdot 1{,}7^{-2}\cdot 10^{3t\cdot\log(1{,}7)} \approx 12{,}87\cdot 10^{0{,}69t}.$

48d $T = 37{,}2\cdot 1{,}7^{3t-2} = 10^{\log(37{,}2)}\cdot\left(10^{\log(1{,}7)}\right)^{3t-2} = 10^{\log(37{,}2)}\cdot 10^{(3t-2)\log(1{,}7)} = 10^{\log(37{,}2)+3t\log(1{,}7)-2\log(1{,}7)} \approx 10^{0{,}69t+1{,}11}.$

```
log(37.2)-2log(1
.7)
       1.109645097
3log(1.7)
       .6913467641
```

49a $N = 18 - 5(6 - 1{,}5^{4t}) = 18 - 30 + 5\cdot 1{,}5^{4t} = -12 + 5\cdot\left(1{,}5^4\right)^t \approx -12 + 5\cdot 5{,}06^t.$

```
1.5^4
       5.0625
```

49b $N = \frac{8^{2t+1}}{4^{t-1}} = \frac{\left(2^3\right)^{2t+1}}{\left(2^2\right)^{t-1}} = \frac{2^{6t+3}}{2^{2t-2}} = 2^{6t+3-(2t-2)} = 2^{6t+3-2t+2} = 2^{4t+5} = 2^{4t}\cdot 2^5 = 32\cdot\left(2^4\right)^t = 32\cdot 16^t.$

```
2^5
       32
2^4
       16
```

49c $K = 150\cdot 1{,}12^{6q+3} = e^{\ln(150)}\cdot\left(e^{\ln(1{,}12)}\right)^{6q+3} = e^{\ln(150)}\cdot e^{(6q+3)\ln(1{,}12)} = e^{\ln(150)+6q\ln(1{,}12)+3\ln(1{,}12)} \approx e^{0{,}68q+5{,}35}$

```
6ln(1.12)
       .6799721118
ln(150)+3ln(1.12
)
       5.35062135
```

50a $T = 27\cdot 0{,}4^t\cdot(3 - 0{,}4^{2t}) = 27\cdot 0{,}4^t\cdot 3 - 27\cdot 0{,}4^{3t} = 81\cdot 0{,}4^t - 27\cdot\left(0{,}4^3\right)^t = 81\cdot 0{,}4^t - 27\cdot 0{,}064^t.$

```
0.4^3
       .064
```

50b $P = 3^{2t+1}\cdot 2^{3t+1} = 3^{2t}\cdot 3^1\cdot 2^{3t}\cdot 2^1 = 6\cdot\left(e^{\ln(3)}\right)^{2t}\cdot\left(e^{\ln(2)}\right)^{3t} = 6\cdot e^{2t\ln(3)}\cdot e^{3t\ln(2)} = 6\cdot e^{t\cdot(2\ln(3)+3\ln(2))} \approx 6e^{4{,}28t}.$

```
2ln(3)+3ln(2)
       4.276666119
```

51 $y = e^{x-1} \Rightarrow \ln(y) = \ln(e^{x-1})$ (ln... en $e^{\cdots}$ zijn elkaars inverse) $= x - 1 \Rightarrow \ln(y) + 1 = x \Rightarrow x = 1 + \ln(y).$

52a
$y = 20\cdot 3^{x-4}$
$\frac{1}{20}y = 3^{x-4}$
${}^{3}\log(0{,}05y) = x - 4$
$x = 4 + {}^{3}\log(0{,}05y)$
$x = 4 + {}^{3}\log(0{,}05) + {}^{3}\log(y)$
$x = 4 + \frac{\log(0{,}05)}{\log(3)} + \frac{\log(y)}{\log(3)}$
$x \approx 1{,}27 + 2{,}10\log(y).$

```
1/20
       .05
4+log(0.05)/log(
3)
       1.273166972
1/log(3)
       2.095903274
```

52b
$y = 0{,}65\cdot 1{,}16^{x-1}$
$\frac{1}{0{,}65}y = 1{,}16^{x-1}$
${}^{1{,}16}\log(\frac{1}{0{,}65}y) = x - 1$
$x = 1 + {}^{1{,}16}\log(\frac{1}{0{,}65}y)$
$x = 1 + {}^{1{,}16}\log(\frac{1}{0{,}65}) + {}^{1{,}16}\log(y)$
$x = 1 + \frac{\ln(\frac{1}{0{,}65})}{\ln(1{,}16)} + \frac{\ln(y)}{\ln(1{,}16)}$
$x \approx 3{,}90 + 6{,}74\ln(y).$

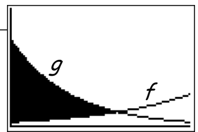
```
1+ln(1/0.65)/ln(
1.16)
       3.902458572
1/ln(1.16)
       6.737636205
```

52c
$N = 250\cdot 10^{2t-3}$
$\frac{1}{250}N = 10^{2t-3}$
$\log(\frac{1}{250}N) = 2t - 3$
$2t = 3 + \log(\frac{1}{250}N)$
$t = 1\frac{1}{2} + \frac{1}{2}\log(\frac{1}{250}N).$

52d
$P = 120\cdot e^{3-q}$
$\frac{1}{120}P = e^{3-q}$
$\ln(\frac{1}{120}P) = 3 - q$
$q = 3 - \ln(\frac{1}{120}P).$

```
Plot1 Plot2 Plot3
\Y1=e^(2X-1)
\Y2=e^(2-3X)
\Y3=
\Y4=
```

```
WINDOW
 Xmin=0
 Xmax=1
 Xscl=0
 Ymin=0
 Ymax=10
 Yscl=0
 Xres=1
```



53a
$f(x) = g(x)$
$e^{2x-1} = e^{2-3x}$
$2x - 1 = 2 - 3x$
$5x = 3$
$x = \frac{3}{5}.$

$O = \int_0^{\frac{3}{5}}\left(g(x) - f(x)\right)\mathrm{d}x = \int_0^{\frac{3}{5}}\left(e^{2-3x} - e^{2x-1}\right)\mathrm{d}x$
$= \left[-\frac{1}{3}\cdot e^{2-3x} - \frac{1}{2}\cdot e^{2x-1}\right]_0^{\frac{3}{5}} = -\frac{1}{3}\cdot e^{\frac{1}{5}} - \frac{1}{2}\cdot e^{\frac{1}{5}} - \left(-\frac{1}{3}\cdot e^2 - \frac{1}{2}\cdot e^{-1}\right)$
$= -\frac{5}{6}\cdot e^{\frac{1}{5}} + \frac{1}{3}\cdot e^2 + \frac{1}{2}\cdot e^{-1} = \frac{1}{3}e^2 - \frac{5}{6}\cdot\sqrt[5]{e} + \frac{1}{2e}.$

53b $h(x) = f(x)\cdot g(x) = e^{2x-1}\cdot e^{2-3x} = e^{1-x}.$
$y = e^{1-x} \Rightarrow \ln(y) = 1 - x \Rightarrow x = 1 - \ln(y).$

53c $y\ln^2(y) - 4y\ln(y) + 5y$ heeft als afgeleide $1\cdot\ln^2(y) + y\cdot 2\ln(y)\cdot\frac{1}{y} - 4\cdot\ln(y) - 4y\cdot\frac{1}{y} + 5$
$= \ln^2(y) + 2\ln(y) - 4\ln(y) - 4 + 5 = 1 - 2\ln(y) + \ln^2(y) = (1 - \ln(y))^2.$
Dus $y\ln^2(y) - 4y\ln(y) + 5y$ is een primitieve van $(1 - \ln(y))^2$.

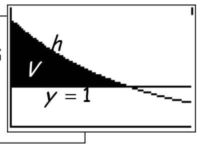
53d $h(0)=e^{1-0}=e^1=e.$

$$I(L)=\int_{y=1}^{y=e}\left(\pi\cdot x^2\right)\mathrm{d}y=\int_1^e\left(\pi\cdot(1-\ln(y))^2\right)\mathrm{d}y=\left[\pi\cdot\left(y\ln^2(y)-4y\ln(y)+5y\right)\right]_1^e$$

$$=\pi\cdot\left(e\ln^2(e)-4e\ln(e)+5e\right)-\left(\pi\cdot\left(\ln^2(1)-4\ln(1)+5\right)\right)=\pi\cdot\left(e\cdot1^2-4e\cdot1+5e\right)-\left(\pi\cdot(0-0+5)\right)=2\pi e-5\pi.$$

```
Plot1 Plot2 Plot3
\Y1=e^(2X-1)
\Y2=e^(2-3X)
\Y3=Y1*Y2
\Y4=1
\Y5=
```

```
WINDOW
 Xmin=0
 Xmax=1.5
 Xscl=0
 Ymin=0
 Ymax=3
 Yscl=0
 Xres=1
```



54 $y=\frac{1}{2}\cdot{}^3\log(x)-2\Rightarrow 2y={}^3\log(x)-4\Rightarrow 2y+4={}^3\log(x)\Rightarrow x=3^{2y+4}=3^{2y}\cdot3^4=3^4\cdot\left(3^2\right)^y=81\cdot9^y.$

```
10^2.15
         141.2537545
10^0.07
         1.174897555
```

55a $\log(N)=2{,}15+0{,}07t\Rightarrow N=10^{2{,}15+0{,}07t}=10^{2{,}15}\cdot10^{0{,}07t}=10^{2{,}15}\cdot\left(10^{0{,}07}\right)^t\approx141\cdot1{,}175^t.$

```
10^2.85
         707.9457844
```

55b $\log(P)=2{,}85+0{,}75\log(q)\Rightarrow P=10^{2{,}85+0{,}75\log(q)}=10^{2{,}85}\cdot10^{0{,}75\log(q)}=10^{2{,}85}\cdot\left(10^{\log(q)}\right)^{0{,}75}\approx708\cdot q^{0{,}75}.$

55c $y=\frac{1}{4}\cdot\ln(5x+2)+3\Rightarrow 4y=\ln(5x+2)+12\Rightarrow 4y-12=\ln(5x+2)\Rightarrow 5x+2=e^{4y-12}\Rightarrow 5x=e^{4y-12}-2\Rightarrow x=\frac{1}{5}e^{4y-12}-\frac{2}{5}.$

55d $\ln(2A+3)=5+\ln(B)\Rightarrow\ln(2A+3)-\ln(B)=5\Rightarrow\ln\left(\frac{2A+3}{B}\right)=5\Rightarrow\frac{2A+3}{B}=e^5\Rightarrow 2A+3=e^5B\Rightarrow 2A=e^5B-3\Rightarrow A=\frac{1}{2}e^5B-1\frac{1}{2}.$

56a $F=2\log(N)-0{,}4\Rightarrow F+0{,}4=2\log(N)\Rightarrow\log(N)=0{,}5F+0{,}2\Rightarrow N=10^{0{,}5F+0{,}2}.$

56b $D=\log(4Q+1)+1\Rightarrow\log(4Q+1)=D-1\Rightarrow 4Q+1=10^{D-1}\Rightarrow 4Q=10^{D-1}-1\Rightarrow Q=\frac{1}{4}\cdot10^{D-1}-\frac{1}{4}.$

56c $K\cdot\sqrt{2}=\ln(2R-3)-\sqrt{6}\Rightarrow\ln(2R-3)=K\cdot\sqrt{2}+\sqrt{6}\Rightarrow 2R-3=e^{K\sqrt{2}+\sqrt{6}}\Rightarrow 2R=e^{K\sqrt{2}+\sqrt{6}}+3\Rightarrow$

$$R=\frac{1}{2}e^{K\sqrt{2}+\sqrt{6}}+1\frac{1}{2}=\frac{1}{2}\cdot\left(e^{\sqrt{2}}\right)^K\cdot e^{\sqrt{6}}+1\frac{1}{2}\approx5{,}8\cdot4{,}1^K+1\frac{1}{2}.$$

```
1/2*e^(√(6))
         5.791217595
e^(√(2))
         4.113250379
```

56d ${}^2\log(3x-1)=-4+{}^2\log(2y+1)\Rightarrow 4={}^2\log(2y+1)-{}^2\log(3x-1)\Rightarrow 4={}^2\log\left(\frac{2y+1}{3x-1}\right)\Rightarrow\frac{2y+1}{3x-1}=2^4=16\Rightarrow$

$$2y+1=48x-16\Rightarrow 2y=48x-17\Rightarrow y=24x-8\frac{1}{2}.$$

57 $\log(a+b)=\log(a)+\log(b)\Rightarrow\log(a+b)=\log(a\cdot b)\Rightarrow a+b=ab\Rightarrow a-ab=-b\Rightarrow a(1-b)=-b\Rightarrow a=-\frac{b}{1-b}=\frac{b}{b-1}.$

```
5.3-1.7log(50)
         2.411750993
10^Ans
         258.0780049
```

58a $D=50$ (cm) $\Rightarrow\log(N)=5{,}3-1{,}7\log(50)\Rightarrow N=10^{5{,}3-1{,}7\log(50)}\approx258$ (bomen per ha).

58b $N=\frac{2000}{8}=250$ (bomen/ha) $\Rightarrow\log(250)=5{,}3-1{,}7\log(D)$ (intersect of) $\Rightarrow$

$$1{,}7\log(D)=5{,}3-\log(250)\Rightarrow\log(D)=\frac{5{,}3-\log(250)}{1{,}7}\Rightarrow D=10^{Ans}\approx51\text{ (cm)}.$$

```
5.3-log(250)
         2.902059991
Ans/1.7
         1.707094113
10^(Ans)
         50.94412561
```

58c $\log(N)=5{,}3-1{,}7\log(D)\Rightarrow 1{,}7\log(D)=5{,}3-\log(N)\Rightarrow\log(D)=\frac{5{,}3}{1{,}7}-\frac{\log(N)}{1{,}7}\Rightarrow$

$$D=10^{\frac{5{,}3}{1{,}7}-\frac{\log(N)}{1{,}7}}=10^{\frac{5{,}3}{1{,}7}}\cdot10^{-\frac{1}{1{,}7}\log(N)}=10^{\frac{5{,}3}{1{,}7}}\cdot\left(10^{\log(N)}\right)^{-\frac{1}{1{,}7}}=10^{\frac{5{,}3}{1{,}7}}\cdot N^{-\frac{1}{1{,}7}}\approx1310\cdot N^{-0{,}59}.$$

```
10^(5.3/1.7)
         1311.133937
-1/1.7
         -.5882352941
```

```
290log(185+100)-
550
         161.9050094
```

59a $G=185$ (cm) $\Rightarrow s=290\log(185+100)-550\approx162$ (cm).

59b $s=210$ (cm) $\Rightarrow 210=290\log(G+100)-550\Rightarrow 760=290\log(G+100)\Rightarrow$

$$\frac{760}{290}=\log(G+100)\Rightarrow G+100=10^{\frac{760}{290}}\Rightarrow G=10^{\frac{760}{290}}-100\approx318\text{ (cm)}.$$

```
10^(760/290)-100
         317.5318937
```

59c $s=290\log(G+100)-550\Rightarrow s+550=290\log(G+100)\Rightarrow\frac{s+550}{290}=\log(G+100)\Rightarrow$

$$G+100=10^{\frac{s+550}{290}}=10^{\frac{1}{290}s}\cdot10^{\frac{550}{290}}\Rightarrow G\approx78{,}8\cdot10^{0{,}00345s}-100.$$

```
1/290
         .0034482759
10^(550/290)
         78.80462816
```

59d $s=180$ (cm) $\Rightarrow G\approx78{,}8\cdot10^{0{,}00345\cdot180}-100\approx229$ en
$s=220$ (cm) $\Rightarrow G\approx78{,}8\cdot10^{0{,}00345\cdot220}-100\approx352.$
Dus de spanwijdten G liggen tussen 229 en 352 cm.

```
78.8*10^(0.00345*
180)-100
         229.2503289
78.8*10^(0.00345*
220)-100
         352.4037722
```

60a $(-1,6)$ invullen in $y=x^2+bx+c$ geeft $6=(-1)^2+b\cdot-1+c\Rightarrow 6=1-b+c\Rightarrow 5=-b+c\Rightarrow -b+c=5$ ❶.
$(4,11)$ invullen in $y=x^2+bx+c$ geeft $11=4^2+b\cdot4+c\Rightarrow 11=16+4b+c\Rightarrow -5=4b+c\Rightarrow 4b+c=-5$ ❷.

60b $\begin{cases}-b+c=5 & ❶\\ 4b+c=-5 & ❷\end{cases}$ −

$-5b=10\Rightarrow b=\frac{10}{-5}=-2$ in ❶ $\Rightarrow 2+c=5\Rightarrow c=3.$

61a $(1, 4\frac{1}{2})$ invullen geeft $\frac{9}{2} = \frac{1+a}{1+b} \Rightarrow 2 + 2a = 9 + 9b \Rightarrow 2a - 9b = 7$ ❶.

$(2, 2\frac{2}{5})$ invullen geeft $\frac{12}{5} = \frac{4+a}{4+b} \Rightarrow 20 + 5a = 48 + 12b \Rightarrow 5a - 12b = 28$ ❷.

$\begin{cases} 2a - 9b = 7 & ❶ \\ 5a - 12b = 28 & ❷ \end{cases} \left|\begin{matrix} 5 \\ 2 \end{matrix}\right. \Rightarrow \begin{cases} 10a - 45b = 35 & ❸ \\ 10a - 24b = 56 & ❹ \end{cases} -$

$-21b = -21 \Rightarrow b = 1$ in ❶ $\Rightarrow 2a - 9 = 7 \Rightarrow 2a = 16 \Rightarrow a = 8$.

61b $(-5, 15)$ invullen geeft $\frac{15}{1} = \frac{25+a}{\sqrt{-5+b}} \Rightarrow 25 + a = 15\sqrt{-5+b}$ ❶.

$(0, 1\frac{2}{3})$ invullen geeft $\frac{5}{3} = \frac{a}{\sqrt{b}} \Rightarrow 3a = 5\sqrt{b} \Rightarrow a = \frac{5}{3}\sqrt{b}$ ❷ (kwadrateren) $\Rightarrow a^2 = \frac{25}{9}b \Rightarrow b = \frac{9}{25}a^2$ ❸.

❸ in ❶ $\Rightarrow 25 + a = 15\sqrt{-5 + \frac{9}{25}a^2}$ (nog eens kwadrateren)

$25^2 + 2 \cdot 25 \cdot a + a^2 = 15^2 \cdot (-5 + \frac{9}{25}a^2)$

$625 + 50a + a^2 = 225 \cdot (-5 + \frac{9}{25}a^2)$

$625 + 50a + a^2 = -1125 + 81a^2$

$-80a^2 + 50a + 1750 = 0$ (hiernaast verder)

$8a^2 - 5a - 175 = 0$

```
5²-4*8*-175
                5625
√(Ans
                  75
```

$D = (-5)^2 - 4 \cdot 8 \cdot -175 = 5625 \Rightarrow \sqrt{D} = 75$

$a = \frac{5+75}{2 \cdot 8} = \frac{80}{2 \cdot 8} = 5 \ \vee \ a = \frac{5-75}{2 \cdot 8} = \frac{-70}{2 \cdot 8} = -\frac{35}{8}$

voldoet niet aan ❷

$a = 5$ in ❸ $\Rightarrow b = \frac{9}{25} \cdot 25 = 9$.

62a $\begin{cases} x^2 + 2y^2 = 18 & ❶ \\ x^2 + y = 17 & ❷ \end{cases} -$

$2y^2 - y = 1 \Rightarrow 2y^2 - y - 1 = 0$ met $D = (-1)^2 - 4 \cdot 2 \cdot -1 = 9 \Rightarrow \sqrt{D} = 3 \Rightarrow y = \frac{1+3}{2 \cdot 2} = \frac{4}{4} = 1 \ \vee \ y = \frac{1-3}{2 \cdot 2} = \frac{-2}{4} = -\frac{1}{2}$

$y = 1$ in ❷ geeft $x^2 + 1 = 17 \Rightarrow x^2 = 16 \Rightarrow x = 4 \ \vee \ x = -4$

$y = -\frac{1}{2}$ in ❷ geeft $x^2 - \frac{1}{2} = 17 \Rightarrow x^2 = 17\frac{1}{2} \Rightarrow x = \sqrt{17\frac{1}{2}} = \sqrt{\frac{35}{2}} = \sqrt{\frac{70}{4}} = \frac{1}{2}\sqrt{70} \ \vee \ x = -\frac{1}{2}\sqrt{70}$.

62b $\begin{cases} x^2 + 2y^2 = 19 & ❶ \\ xy = 3 & ❷ \end{cases} \Rightarrow \begin{cases} x^2 + 2y^2 = 19 & ❶ \\ y = \frac{3}{x} & ❸ \end{cases}$

❸ in ❶ $\Rightarrow x^2 + 2\left(\frac{3}{x}\right)^2 = 19$

$x^2 + 2 \cdot \frac{9}{x^2} = 19$ (vermenigvuldigen met x^2)

$x^4 - 19x^2 + 18 = 0$

$(x^2 - 1)(x^2 - 18) = 0$ (hiernaast verder)

$x^2 = 1 \ \vee \ x^2 = 18$

$x = 1 \ \vee \ x = -1 \ \vee \ x = \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2} \ \vee \ x = -3\sqrt{2}$

$x = 1$ in ❸ geeft $y = \frac{3}{1} = 3$

$x = -1$ in ❸ geeft $y = \frac{3}{-1} = -3$

$x = 3\sqrt{2}$ in ❸ geeft $y = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$

$x = -3\sqrt{2}$ in ❸ geeft $y = \frac{3}{-3\sqrt{2}} = -\frac{1}{2}\sqrt{2}$.

62c $\begin{cases} a + b = 8 & ❶ \\ 2^a + 2^{b-1} = 24 & ❷ \end{cases} \Rightarrow \begin{cases} b = 8 - a & ❸ \\ 2^a + 2^{b-1} = 24 & ❷ \end{cases}$

❸ in ❷ $\Rightarrow 2^a + 2^{8-a-1} = 24$

$2^a + 2^{7-a} = 24$

$2^a + \frac{2^7}{2^a} = 24$ (vermenigvuldigen met 2^a)

$\left(2^a\right)^2 - 24 \cdot 2^a + 2^7 = 0$ (hiernaast verder)

```
2^7
                 128
```

$\left(2^a\right)^2 - 24 \cdot 2^a + 128 = 0$

$(2^a - 8)(2^a - 16) = 0$

$2^a = 8 = 2^3 \ \vee \ 2^a = 16 = 2^4$

$a = 3 \ \vee \ a = 4$

$a = 3$ in ❸ geeft $b = 8 - 3 = 5$

$a = 4$ in ❸ geeft $b = 8 - 4 = 4$.

62d $\begin{cases} a + 2b = 14 & ❶ \\ {}^2\log(a) + {}^2\log(b-1) = 4 & ❷ \end{cases} \Rightarrow \begin{cases} a = 14 - 2b & ❸ \\ {}^2\log(a \cdot (b-1)) = 4 & ❹ \end{cases} \Rightarrow \begin{cases} a = 14 - 2b & ❸ \\ a \cdot (b-1) = 2^4 = 16 & ❺ \end{cases}$

❸ in ❺ $\Rightarrow (14 - 2b) \cdot (b - 1) = 16$

$14b - 14 - 2b^2 + 2b - 16 = 0$

$-2b^2 + 16b - 30 = 0$

$b^2 - 8b + 15 = 0$ (hiernaast verder)

$(b-3)(b-5) = 0$

$b = 3 \ \vee \ b = 5$

$b = 3$ in ❸ geeft $a = 14 - 6 = 8$

$b = 5$ in ❸ geeft $a = 14 - 10 = 4$.

63a $f_{0,0}(x) = \frac{3^{x+0}+1}{3^{x+0}-1} = \frac{3^x+1}{3^x-1}$. Nu is $f_{0,0}(a) = \frac{3^a+1}{3^a-1}$ en $f_{0,0}(-a) = \frac{3^{-a}+1}{3^{-a}-1} = \frac{3^{-a}+1}{3^{-a}-1} \cdot \frac{3^a}{3^a} = \frac{3^0+3^a}{3^0-3^a} = \frac{1+3^a}{1-3^a} = -\frac{3^a+1}{3^a-1}$.

Voor elke a is $f_{0,0}(a) = -f_{0,0}(-a) \Rightarrow$ de grafiek van $f_{0,0}$ is puntsymmetrisch in de oorsprong.

63b $f_{-1,-1}(x) = \frac{3^{x-1}+1}{3^{x-1}-1} = \frac{3^{x-1}+1}{3^{x-1}-1} \cdot \frac{3}{3} = \frac{3^x+3}{3^x-3}$.

63c $(3, \frac{1}{4})$ invullen in $f_{p,q}(x) = \frac{3^{x+p}+1}{3^{x+q}-1}$ geeft $\frac{1}{4} = \frac{3^{3+p}+1}{3^{3+q}-1} \Rightarrow 3^{3+q} - 1 = 4 \cdot 3^{3+p} + 4 \Rightarrow 3^{3+q} = 4 \cdot 3^{3+p} + 5$ ❶.

$(5, \frac{1}{8})$ invullen geeft $\frac{1}{8} = \frac{3^{5+p}+1}{3^{5+q}-1} \Rightarrow 3^{5+q} - 1 = 8 \cdot 3^{5+p} + 8 \Rightarrow 3^{5+q} = 8 \cdot 3^{5+p} + 9$ (delen door 3^2) $\Rightarrow 3^{3+q} = 8 \cdot 3^{3+p} + 1$ ❷.

❶ in ❷ $\Rightarrow 4 \cdot 3^{3+p} + 5 = 8 \cdot 3^{3+p} + 1 \Rightarrow -4 \cdot 3^{3+p} = -4 \Rightarrow 3^{3+p} = 1 = 3^0 \Rightarrow 3 + p = 0 \Rightarrow p = -3$.

$p = -3$ in ❶ $\Rightarrow 3^{3+q} = 4 \cdot 3^0 + 5 = 4 \cdot 1 + 5 = 9 = 3^2 \Rightarrow 3 + q = 2 \Rightarrow q = -1$.

64a $f(x)=\frac{4}{x+1}+\frac{5}{x+2}=4\cdot\frac{1}{x+1}+5\cdot\frac{1}{x+2}\Rightarrow F(x)=4\cdot\ln|x+1|+5\cdot\ln|x+2|.$

64b $f(x)=\frac{4}{x+1}+\frac{5}{x+2}=\frac{4}{x+1}\cdot\frac{x+2}{x+2}+\frac{5}{x+2}\cdot\frac{x+1}{x+1}=\frac{4(x+2)+5(x+1)}{(x+1)(x+2)}=\frac{4x+8+5x+5}{(x+1)(x+2)}=\frac{9x+13}{(x+1)(x+2)}.$

64c $g(x)=\frac{9x+13}{x^2+3x+2}=\frac{9x+13}{(x+1)(x+2)}=f(x)\Rightarrow G(x)=F(x)=4\cdot\ln|x+1|+5\cdot\ln|x+2|.$

65a $f(x)=\frac{5x-14}{x^2-6x+8}=\frac{3}{x-4}+\frac{2}{x-2}$ (zie de berekening hieronder) $\Rightarrow F(x)=3\ln|x-4|+2\ln|x-2|+c.$

$f(x)=\frac{5x-14}{x^2-6x+8}=\frac{5x-14}{(x-4)(x-2)}=\frac{a}{x-4}+\frac{b}{x-2}=\frac{a(x-2)}{(x-4)(x-2)}+\frac{b(x-4)}{(x-4)(x-2)}=\frac{ax-2a+bx-4b}{(x-4)(x-2)}=\frac{(a+b)x-2a-4b}{(x-4)(x-2)}$

$\begin{cases} a+b=5 & ❶ \\ -2a-4b=-14 & ❷ \end{cases}\begin{vmatrix}2\\1\end{vmatrix}\Rightarrow\begin{cases} 2a+2b=10 & ❸ \\ -2a-4b=-14 & ❷ \end{cases}+$

$-2b=-4\Rightarrow b=2$ in ❶ $\Rightarrow a+2=5\Rightarrow a=3.$

65b $g(x)=\frac{x+11}{x^2+4x+3}=-\frac{4}{x+3}+\frac{5}{x+1}$ (zie de berekening hieronder) $\Rightarrow G(x)=-4\ln|x+3|+5\ln|x+1|+c.$

$g(x)=\frac{x+11}{x^2+4x+3}=\frac{x+11}{(x+3)(x+1)}=\frac{a}{x+3}+\frac{b}{x+1}=\frac{a(x+1)}{(x+3)(x+1)}+\frac{b(x+3)}{(x+3)(x+1)}=\frac{ax+a+bx+3b}{(x+3)(x+1)}=\frac{(a+b)x+a+3b}{(x+3)(x+1)}$

$\begin{cases} a+b=1 & ❶ \\ a+3b=11 & ❷ \end{cases}-$

$-2b=-10\Rightarrow b=5$ in ❶ $\Rightarrow a+5=1\Rightarrow a=-4.$

65c $h(x)=\frac{3x+24}{x^2+7x+10}=-\frac{3}{x+5}+\frac{6}{x+2}$ (zie de berekening hieronder) $\Rightarrow H(x)=-3\ln|x+5|+6\ln|x+2|+c.$

$h(x)=\frac{3x+24}{x^2+7x+10}=\frac{3x+24}{(x+5)(x+2)}=\frac{a}{x+5}+\frac{b}{x+2}=\frac{a(x+2)}{(x+5)(x+2)}+\frac{b(x+5)}{(x+5)(x+2)}=\frac{ax+2a+bx+5b}{(x+5)(x+2)}=\frac{(a+b)x+2a+5b}{(x+5)(x+2)}$

$\begin{cases} a+b=3 & ❶ \\ 2a+5b=24 & ❷ \end{cases}\begin{vmatrix}2\\1\end{vmatrix}\Rightarrow\begin{cases} 2a+2b=6 & ❸ \\ 2a+5b=24 & ❷ \end{cases}-$

$-3b=-18\Rightarrow b=6$ in ❶ $\Rightarrow a+6=3\Rightarrow a=-3.$

65d $j(x)=\frac{x}{x^2+7x+12}=\frac{4}{x+4}-\frac{3}{x+3}$ (zie de berekening hieronder) $\Rightarrow J(x)=4\ln|x+4|-3\ln|x+3|+c.$

$j(x)=\frac{x}{x^2+7x+12}=\frac{x}{(x+4)(x+3)}=\frac{a}{x+4}+\frac{b}{x+3}=\frac{a(x+3)}{(x+4)(x+3)}+\frac{b(x+4)}{(x+4)(x+3)}=\frac{ax+3a+bx+4b}{(x+4)(x+3)}=\frac{(a+b)x+3a+4b}{(x+4)(x+3)}$

$\begin{cases} a+b=1 & ❶ \\ 3a+4b=0 & ❷ \end{cases}\begin{vmatrix}3\\1\end{vmatrix}\Rightarrow\begin{cases} 3a+3b=3 & ❸ \\ 3a+4b=0 & ❷ \end{cases}-$

$-b=3\Rightarrow b=-3$ in ❶ $\Rightarrow a-3=1\Rightarrow a=4.$

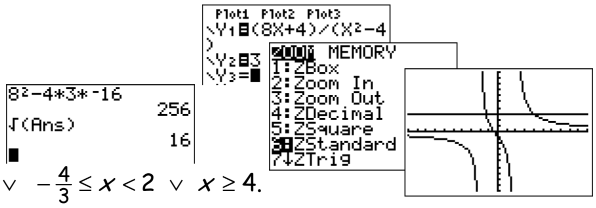
66a $f(x)=\frac{8x+4}{x^2-4}=\frac{3}{1}\quad (x^2\neq 4\Rightarrow x\neq\pm 2)$

$3x^2-12=8x+4$

$3x^2-8x-16=0$

$D=(-8)^2-4\cdot 3\cdot -16=256\Rightarrow\sqrt{D}=16$

$x=\frac{8+16}{2\cdot 3}=\frac{24}{6}=4\ \vee\ x=\frac{8-16}{2\cdot 3}=\frac{-8}{6}=-\frac{4}{3}.$



$f(x)\leq 3$ (zie hierboven en een plot) $\Rightarrow x<-2\ \vee\ -\frac{4}{3}\leq x<2\ \vee\ x\geq 4.$

66b $f(x)=\frac{8x+4}{x^2-4}\Rightarrow f'(x)=\frac{(x^2-4)\cdot 8-(8x+4)\cdot 2x}{(x^2-4)^2}=\frac{8x^2-32-16x^2-8x}{(x^2-4)^2}=\frac{-8x^2-8x-32}{(x^2-4)^2}.$

$f(0)=\frac{0+4}{0-4}=-1$ en $f'(0)=\frac{0-0-32}{(0-4)^2}=\frac{-32}{16}=-2.$

k: $y=-2x+b$ door $(0,\ -1)\Rightarrow k$: $y=-2x-1.$

66c $f(x)=\frac{8x+4}{x^2-4}=\frac{4}{3}$

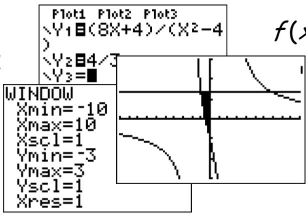
$4x^2-16=24x+12$

$4x^2-24x-28=0$

$x^2-6x-7=0$

$(x-7)(x+1)=0$

$x=7\ \vee\ x=-1.$



$f(x)=\frac{8x+4}{x^2-4}=\frac{8x+4}{(x-2)(x+2)}=\frac{a}{x-2}+\frac{b}{x+2}=\frac{a(x+2)}{(x-2)(x+2)}+\frac{b(x-2)}{(x-2)(x+2)}$

$=\frac{ax+2a+bx-2b}{(x-2)(x+2)}=\frac{(a+b)x+2a-2b}{(x-2)(x+2)}.$

$\begin{cases} a+b=8 & ❶ \\ 2a-2b=4 & ❷ \end{cases}\begin{vmatrix}2\\1\end{vmatrix}\Rightarrow\begin{cases} 2a+2b=16 & ❸ \\ 2a-2b=4 & ❷ \end{cases}+$

$4a=20\Rightarrow a=5$ in ❶ $\Rightarrow 5+b=8\Rightarrow b=3.$

$O(V)=\int_{-1}^{0}\left(1\tfrac{1}{3}-f(x)\right)dx=\int_{-1}^{0}\left(1\tfrac{1}{3}-(\tfrac{5}{x-2}+\tfrac{3}{x+2})\right)dx=\int_{-1}^{0}\left(1\tfrac{1}{3}-\tfrac{5}{x-2}-\tfrac{3}{x+2}\right)dx=\left[1\tfrac{1}{3}x-5\ln|x-2|-3\ln|x+2|\right]_{-1}^{0}$

$=0-5\ln(2)-3\ln(3)-\left(-1\tfrac{1}{3}-5\ln(3)-3\ln(1)\right)=-8\ln(2)+1\tfrac{1}{3}+5\ln(3).$

67a $e^{x-p}=6$ ❶ invullen in $-2p\cdot e^{x-p}=24$ ❷ geeft $-2p\cdot 6=24.$

67b $-2p\cdot 6=24\Rightarrow -12p=24\Rightarrow p=-2$ in ❶ $\Rightarrow e^{x+2}=6\Rightarrow x+2=\ln(6)\Rightarrow x=-2+\ln(6).$

68a $p\sqrt{x^2+4}=4$ ❷ invullen in $px\sqrt{x^2+4}=x^2+3$ ❶ geeft $4x=x^2+3$
$x^2-4x+3=0$
$(x-3)(x-1)=0$
$x=3 \vee x=1$ (hiernaast verder)
$x=3$ in ❷ $\Rightarrow p\sqrt{3^2+4}=4 \Rightarrow p=\frac{4}{\sqrt{13}}=\frac{4}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}}=\frac{4}{13}\sqrt{13}$ en
$x=1$ in ❷ $\Rightarrow p\sqrt{1^2+4}=4 \Rightarrow p=\frac{4}{\sqrt{5}}=\frac{4}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{4}{5}\sqrt{5}$.

68b $pe^{x^2-1}=x^2+1$ ❶ invullen in $pxe^{x^2-1}=x^3+x^2-6$ ❷ geeft $x(x^2+1)=x^3+x^2-6$
$x^3+x=x^3+x^2-6$
$x^2-x-6=0$
$(x-3)(x+2)=0$
$x=3 \vee x=-2$ (hiernaast verder)
$x=3$ in ❶ $\Rightarrow pe^{3^2-1}=3^2+1 \Rightarrow pe^8=10 \Rightarrow \frac{10}{e^8}$ en
$x=-2$ in ❶ $\Rightarrow pe^{4-1}=4+1 \Rightarrow pe^3=5 \Rightarrow \frac{5}{e^3}$.

68c $2+a\ln(x+1)=x \Rightarrow a\ln(x+1)=x-2$ ❶ invullen in $2ax\ln(x+1)=48$ (2) geeft $2x\cdot(x-2)=48$
$2x^2-4x=48$
$x^2-2x-24=0$
$(x-6)(x+4)=0$
$x=6 \vee x=-4$ (hiernaast verder)
$x=6$ in ❶ $\Rightarrow a\ln(6+1)=6-2 \Rightarrow a=\frac{4}{\ln(7)}$ en
$x=-4$ in ❶ $\Rightarrow a\ln(-4+1)=\ldots$ heeft geen oplossing.

68d $x+\frac{x+p}{2x+3}=2 \Rightarrow \frac{x+p}{2x+3}=2-x$ ❶ invullen in $\sqrt{\frac{x+p}{2x+3}}=x$ ❷ geeft $\sqrt{2-x}=x$ (* kwadrateren)
$2-x=x^2$
$x^2+x-2=0$
$(x+2)(x-1)=0$
$x=-2$ (voldoet niet aan *) $\vee$ $x=1$ in ❶ $\Rightarrow \frac{1+p}{2+3}=2-1 \Rightarrow 1+p=5\cdot 1 \Rightarrow p=5-1=4$.

69 $f_p(x)=e^{x^2+p}+x \Rightarrow f_p{}'(x)=e^{x^2+p}\cdot 2x+1$.
x-as raken $\Rightarrow f_p(x)=0$ (op de x-as komen) $\wedge$ $f_p{}'(x)=0$ (horizontale raaklijn)
$e^{x^2+p}+x=0 \Rightarrow e^{x^2+p}=-x$ ❶ invullen in $e^{x^2+p}\cdot 2x+1=0$ ❷ geeft $-x\cdot 2x+1=0$
$-2x^2+1=0$
$2x^2=1$
$x^2=\frac{1}{2}$
$x=\sqrt{\frac{1}{2}}=\frac{\sqrt{1}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}=\frac{1}{2}\sqrt{2} \vee x=-\sqrt{\frac{1}{2}}=-\frac{1}{2}\sqrt{2}$ (hierboven verder)

$x=\frac{1}{2}\sqrt{2}=\sqrt{\frac{1}{2}}$ in ❶ $\Rightarrow e^{\frac{1}{2}+p}=-\frac{1}{2}\sqrt{2}$ heeft geen oplossing en
$x=-\frac{1}{2}\sqrt{2}=-\sqrt{\frac{1}{2}}$ in ❶ $\Rightarrow e^{\frac{1}{2}+p}=\frac{1}{2}\sqrt{2} \Rightarrow \frac{1}{2}+p=\ln(\frac{1}{2}\sqrt{2}) \Rightarrow p=-\frac{1}{2}+\ln(\frac{1}{2}\sqrt{2})$.

Dus $x=-\frac{1}{2}\sqrt{2}$ en het raakpunt is $\left(-\frac{1}{2}\sqrt{2},\ 0\right)$.

70 $f_p(x)=\sqrt{x^2+p} \Rightarrow f_p{}'(x)=\frac{1}{2\sqrt{x^2+p}}\cdot 2x=\frac{x}{\sqrt{x^2+p}}$.
$f_p(x)=5x+5$ ($y=5x+5$ snijden) $\wedge$ $f_p{}'(x)\cdot 5=-1$ (loodrecht op $y=5x+5$)
$\sqrt{x^2+p}=5x+5$ ❶ invullen in $\frac{x}{\sqrt{x^2+p}}\cdot 5=-1$ ❷ geeft $\frac{x}{5x+5}\cdot 5=-1$
$\frac{x}{x+1}=\frac{-1}{1}$
$x=-x-1$
$2x=-1$
$x=-\frac{1}{2}$ in ❶ $\Rightarrow \sqrt{\frac{1}{4}+p}=5\cdot-\frac{1}{2}+5 \Rightarrow \sqrt{\frac{1}{4}+p}=2\frac{1}{2}$ (* kwadrateren) $\Rightarrow \frac{1}{4}+p=\frac{25}{4} \Rightarrow p=\frac{24}{4}=6$ (voldoet aan *).

71 $f_p(x)=\ln(p-x^2) \Rightarrow f_p{}'(x)=\frac{1}{p-x^2}\cdot -2x=-\frac{2x}{p-x^2}$ en $g_q(x)=-x^2+q \Rightarrow g_q{}'(x)=-2x$.
$f_p(x)=g_q(x) \wedge f_p{}'(x)=g_q{}'(x)$
$\ln(p-x^2)=-x^2+q$ ❶ $\wedge$ $-\frac{2x}{p-x^2}=-2x$ ❷
$\ln(p-x^2)=-x^2+q \wedge \left(x=0 \vee p-x^2=1\right)$
$\left(\ln(p-x^2)=-x^2+q \wedge x=0\right) \vee \left(\ln(p-x^2)=-x^2+q \wedge p-x^2=1\right)$
$\left(\ln(p)=q\right) \vee \left(\ln(1)=-x^2+q\right)$
$\left(p=e^q\right) \vee \left(x^2=q \text{ in ❶}\right) \Rightarrow p=e^q \vee \ln(p-q)=0 \Rightarrow p=e^q \vee p-q=e^0=1 \Rightarrow p=e^q \vee p=1+q$.

Diagnostische toets

D1a $(x^2-4)^2=(-x+2)^2$
$x^2-4=-x+2 \vee x^2-4=x-2$
$x^2+x-6=0 \vee x^2-x-2=0$
$(x+3)(x-2)=0 \vee (x-2)(x+1)=0$
$x=-3 \vee x=2 \vee x=2 \vee x=-1$
$x=-3 \vee x=2 \vee x=-1.$

D1b $e^x \cdot \sin(x)=5\sin(x)$
$\sin(x)=0 \vee e^x=5$
$x=k\cdot\pi \vee x=\ln(5).$

D1c $\frac{\ln(x)}{\ln(x)-1}=\frac{\ln(x)+2}{\ln(x)-4}$
(stel tijdelijk $\ln(x)=t$)
$\frac{t}{t-1}=\frac{t+2}{t-4}$
$t(t-4)=(t-1)(t+2)$
$t^2-4t=t^2+t-2$
$-5t=-2$
$t=\ln(x)=\frac{2}{5}\Rightarrow x=e^{\frac{2}{5}}=\sqrt[5]{e^2}.$

D2a $((x-\sqrt{2})(x+\sqrt{2}))^2=(x^2-2)^2=x^4-4x^2+4.$

D2b $(2x+3)^3=(2x+3)(2x+3)^2=(2x+3)(4x^2+12x+9)^2=8x^3+24x^2+18x+12x^2+36x+27=8x^3+36x^2+54x+27.$

D2c $\frac{x^4-16}{x^2-4}\ (x^2\neq4\Rightarrow x\neq2\wedge x\neq-2)=\frac{(x^2+4)(x^2-4)}{x^2-4}=x^2+4\ (x\neq2\wedge x\neq-2).$

D3a $y=\frac{6}{x+1}\cdot\frac{2}{x-1}\ (x\neq-1\wedge x\neq1)=\frac{12}{(x+1)(x-1)}.$

D3b $y=\frac{\frac{x-1}{x+1}}{2x^2}\ (x\neq0\wedge x\neq-1)=\frac{x-1}{x+1}\cdot\frac{1}{2x^2}=\frac{x-1}{2x^2(x+1)}.$

D3c $y=\frac{x-1}{\frac{x+1}{2x^2}}\ (x\neq0\wedge x\neq-1)=(x-1)\cdot\frac{2x^2}{x+1}=\frac{2x^2(x-1)}{x+1}.$

D3d $y=\frac{2e^x}{e^x-1}+\frac{e^{-x}}{e^x+1}\ (e^x\neq1\Rightarrow x\neq0)=\frac{2e^x(e^x+1)+e^{-x}(e^x-1)}{(e^x-1)(e^x+1)}=\frac{2e^{2x}+2e^x+1-e^{-x}}{(e^x-1)(e^x+1)}\cdot\frac{e^x}{e^x}=\frac{2e^{3x}+2e^{2x}+e^x-1}{e^x(e^x-1)(e^x+1)}.$

D3e $y=\frac{e^{-x}}{x+1}\cdot3e^{-x}\ (x\neq-1)=\frac{3e^{-2x}}{x+1}\cdot\frac{e^{2x}}{e^{2x}}=\frac{3}{e^{2x}(x+1)}.$

D3f $y=\frac{x+\frac{x}{x-1}}{x+1}\ (x\neq1\wedge x\neq-1)=\frac{x(x-1)+x}{(x+1)(x-1)}=\frac{x^2-x+x}{2x^2}=\frac{x^2}{(x+1)(x-1)}.$

D4a $\frac{1}{b}+\frac{1}{v}-\frac{1}{f}=0\ (b\neq0\wedge v\neq0\wedge f\neq0)\Rightarrow\frac{1}{v}=\frac{1}{f}-\frac{1}{b}\Rightarrow\frac{1}{v}=\frac{b}{bf}-\frac{f}{bf}=\frac{b-f}{bf}\Rightarrow v=\frac{bf}{b-f}\ (b\neq0\wedge v\neq0\wedge f\neq0\wedge b\neq f).$

D4b $\frac{3}{p}+\frac{4}{q}=6\ (p\neq0\wedge q\neq0)\Rightarrow\frac{3}{p}=6-\frac{4}{q}\Rightarrow\frac{3}{p}=\frac{6q}{q}-\frac{4}{q}=\frac{6q-4}{q}\Rightarrow\frac{p}{3}=\frac{q}{6q-4}\Rightarrow p=\frac{3q}{6q-4}\ (p\neq0\wedge q\neq0\wedge q\neq\frac{2}{3}).$

D5a $y=\frac{2}{\sqrt{x}}+\frac{4}{3}\sqrt{x}=\frac{2}{\sqrt{x}}\cdot\frac{3}{3}+\frac{4\sqrt{x}}{3}\cdot\frac{\sqrt{x}}{\sqrt{x}}=\frac{6}{3\sqrt{x}}+\frac{4x}{3\sqrt{x}}=\frac{6+4x}{3\sqrt{x}}\ (x>0).$

D5b $A=\sqrt{8p}-\sqrt{\frac{32}{p}}=\sqrt{4\cdot2p}-\sqrt{\frac{16\cdot2}{p}\cdot\frac{p}{p}}=2\cdot\sqrt{2p}-\frac{4}{p}\cdot\sqrt{2p}=\left(2-\frac{4}{p}\right)\cdot\sqrt{2p}\ (p>0).$

D5c $y=\frac{3}{\sqrt{x^2+1}}-\sqrt{x^2+1}=\frac{3}{\sqrt{x^2+1}}-\frac{x^2+1}{\sqrt{x^2+1}}=\frac{3-x^2-1}{\sqrt{x^2+1}}=\frac{2-x^2}{\sqrt{x^2+1}}.$

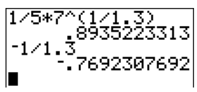
D6a $x\sqrt{2y-1}-\frac{2}{\sqrt{x}}=0\ (x>0\wedge y\geq\frac{1}{2})$
$x\sqrt{2y-1}=\frac{2}{\sqrt{x}}$
$\sqrt{2y-1}=\frac{2}{x\sqrt{x}}$ (kwadrateren)
$2y-1=\frac{4}{x^3}$
$2y=\frac{4}{x^3}+1\Rightarrow y=\frac{2}{x^3}+\frac{1}{2}.$

D6b $A=2\sqrt{p-2A}\ (p\geq2A)$
$A^2=4(p-2A)$
$p-2A=\frac{A^2}{4}$
$p=\frac{A^2}{4}+2A.$

D7a $y=\left(\frac{2}{x\cdot\sqrt[5]{x^2}}\right)^2\cdot x\sqrt{x}=\left(\frac{2}{x^{1\frac{2}{5}}}\right)^2\cdot x^{1\frac{1}{2}}=\frac{4}{x^{2\frac{4}{5}}}\cdot x^{1\frac{1}{2}}=4\cdot x^{-2\frac{4}{5}}\cdot x^{1\frac{1}{2}}=4x^{-1\frac{3}{10}}.$

D7b $y=\left(2x^{0,7}\right)^3\cdot\left(81x^3\right)^{0,5}=2^3\cdot x^{2,1}\cdot81^{\frac{1}{2}}\cdot x^{1,5}=8\cdot x^{2,1}\cdot9\cdot x^{1,5}=72x^{3,6}.$

D8a $P=7\cdot(5A)^{-1,3}\Rightarrow P=\frac{7}{(5A)^{1,3}}\Rightarrow(5A)^{1,3}=\frac{7}{P}\Rightarrow5A=\left(\frac{7}{P}\right)^{\frac{1}{1,3}}=\frac{7^{\frac{1}{1,3}}}{P^{\frac{1}{1,3}}}\Rightarrow A=\frac{1}{5}\cdot7^{\frac{1}{1,3}}\cdot P^{-\frac{1}{1,3}}\approx0,89P^{-0,77}.$



D8b $x=2t\cdot\sqrt[3]{t}=2t\cdot t^{\frac{1}{3}}=2t^{1\frac{1}{3}}$ invullen in $y=3x^2\cdot\sqrt{x}=3x^{2\frac{1}{2}}$ geeft $y=3\left(2t^{\frac{4}{3}}\right)^{\frac{5}{2}}=3\cdot2^{2\frac{1}{2}}\cdot t^{\frac{10}{3}}=3\cdot4\sqrt{2}\cdot t^{3\frac{1}{3}}=12\sqrt{2}\cdot t^{3\frac{1}{3}}.$

D9a $N = 2^{t-1} \cdot \left(\frac{1}{4}\right)^{3t+2} = 2^{t-1} \cdot \left(2^{-2}\right)^{3t+2} = 2^{t-1} \cdot 2^{-6t-4} = 2^{-5t-5} = 2^{-5t} \cdot 2^{-5} = \left(2^{-5}\right)^{t} \cdot \frac{1}{32} = \frac{1}{32} \cdot \left(\frac{1}{32}\right)^{t}$.

```
2^5
                 32
```

D9b $N = 2 \cdot 3^{2t-1} = 2 \cdot 3^{2t} \cdot 3^{-1} = 2 \cdot \left(3^{2}\right)^{t} \cdot \frac{1}{3} = \frac{2}{3} \cdot 9^{t}$.

D10a $y = 1200 \cdot 2^{x-1} = 1200 \cdot 2^{x} \cdot 2^{-1} = 1200 \cdot \left(e^{\ln(2)}\right)^{x} \cdot \frac{1}{2} = 600 \cdot e^{x\ln(2)} \approx 600 \cdot e^{0{,}69x}$.

```
ln(2)
        .6931471806
```

D10b $y = \frac{1}{1100} \cdot \left(\frac{1}{2}\right)^{x-6} = \frac{1}{1100} \cdot \left(2^{-1}\right)^{x-6} = \frac{1}{1100} \cdot 2^{-x} \cdot 2^{6} = \frac{1}{1100} \cdot \left(10^{\log(2)}\right)^{-x} \cdot 64 = \frac{64}{1100} \cdot 10^{-x\log(2)} \approx 0{,}06 \cdot 10^{-0{,}30x}$.

```
2^6
                 64
Ans/1100
        .0581818182
-log(2)
       -.3010299957
```

D10c $y = 10 \cdot 2{,}21^{2x+3} = e^{\ln(10)} \cdot \left(e^{\ln(2{,}21)}\right)^{2x+3} = e^{\ln(10)+2x\ln(2{,}21)+3\ln(2{,}21)} \approx e^{1{,}59x+4{,}68}$.

```
ln(10)+3ln(2.21)
         4.68156264
2ln(2.21)
        1.585985031
```

D11a
$N = 10 \cdot 2{,}21^{2x+3}$
$\frac{1}{10}N = 2{,}21^{2x+3}$
$\log(\frac{1}{10}N) = \log(2{,}21^{2x+3})$
$\log(\frac{1}{10}) + \log(N) = (2x+3)\log(2{,}21)$
$\frac{\log(10^{-1}) + \log(N)}{\log(2{,}21)} = 2x + 3$
$2x = \frac{-1+\log(N)}{\log(2{,}21)} - 3$
$x = \frac{-1}{2\log(2{,}21)} + \frac{\log(N)}{2\log(2{,}21)} - \frac{3}{2}$
$x \approx -2{,}95 + 1{,}45\log(N)$.

```
-1/(2log(2.21))-
3/2
       -2.951832803
1/(2log(2.21))
        1.451832803
```

D11b
$y = \frac{1}{3} \cdot 2^{5x-6}$
$3y = 2^{5x-6}$
$\ln(3y) = \ln(2^{5x-6})$
$\ln(3) + \ln(y) = (5x-6)\ln(2)$
$\frac{\ln(3)+\ln(y)}{\ln(2)} = 5x - 6$
$5x = \frac{\ln(3)+\ln(y)}{\ln(2)} + 6$
$x = \frac{\ln(3)}{5\ln(2)} + \frac{\ln(y)}{5\ln(2)} + \frac{6}{5}$
$x \approx 1{,}52 + 0{,}29\ln(y)$.

```
ln(3)/(5ln(2))+6
/5
          1.5169925
1/(5ln(2))
        .2885390082
```

D12a $\log(N) = 2{,}1 + 0{,}1t \Rightarrow N = 10^{2{,}1+0{,}1t} = 10^{2{,}1} \cdot \left(10^{0{,}1}\right)^{t} \approx 126 \cdot 1{,}259^{t}$.

```
10^2.1
        125.8925412
10^0.1
        1.258925412
```

```
10^1.7
        50.11872336
```

D12b $\log(p) = 1{,}7 - 0{,}02\log(q) \Rightarrow \log(p) = \log\left(10^{1{,}7}\right) + \log(q^{-0{,}02}) = \log\left(10^{1{,}7} \cdot q^{-0{,}02}\right) \Rightarrow p = 10^{1{,}7} \cdot q^{-0{,}02} \approx 50{,}12 \cdot q^{-0{,}02}$.

D12c $\ln(y+2) = 2 + \ln(x-4) \Rightarrow \ln(y+2) = \ln\left(e^{2}\right) + \ln(x-4) = \ln\left(e^{2}(x-4)\right) \Rightarrow y + 2 = e^{2}x - 4e^{2} \Rightarrow$
$y = e^{2}x - 4e^{2} - 2 \approx 7{,}39x - 31{,}56$.

```
e²
        7.389056099
-4e²-2
        -31.5562244
```

D12d $K = 3\ln(2F-1) + 4 \Rightarrow K - 4 = 3\ln(2F-1) \Rightarrow \frac{K-4}{3} = \ln(2F-1) \Rightarrow 2F - 1 = e^{\frac{1}{3}K - \frac{4}{3}} \Rightarrow 2F = 1 + e^{\frac{1}{3}K - \frac{4}{3}} \Rightarrow F = \frac{1}{2} + \frac{1}{2}e^{\frac{1}{3}K - \frac{4}{3}}$.

D13a
$$\begin{cases} 2x^2 + 6y^2 = 20 & ❶ \\ 2x^2 + 6y = 8 & ❷ \end{cases} \quad -$$
$6y^2 - 6y = 12$
$y^2 - y = 2$ (hiernaast verder)

$y^2 - y - 2 = 0$
$(y-2)(y+1) = 0$
$y = 2$ in ❷ $\Rightarrow 2x^2 + 12 = 8 \Rightarrow 2x^2 = -4 \Rightarrow x^2 = -...$ geen oplossing
$\underline{y = -1}$ in ❷ $\Rightarrow 2x^2 - 6 = 8 \Rightarrow 2x^2 = 14 \Rightarrow x^2 = 7 \Rightarrow \underline{x = 7 \vee x = -7}$.

D13b $x + y = 4 \Rightarrow x = 4 - y$ ❶ invullen in $2^{x} + 2^{-y} = 17$ ❷ geeft $2^{4-y} + 2^{-y} = 17$
$2^{4-y} + 2^{-y} = 17$
$2^{4} \cdot 2^{-y} + 2^{-y} = 17$
$16 \cdot 2^{-y} + 1 \cdot 2^{-y} = 17$
$17 \cdot 2^{-y} = 17$ (hiernaast verder)

$2^{-y} = 1 = 2^{0}$
$-y = 0$
$y = 0$
$\underline{y = 0}$ in ❶ $\Rightarrow x + 0 = 4 \Rightarrow \underline{x = 4}$.

D13c $2a - 7 = 2\sqrt{b-2}$ (kwadrateren) $\Rightarrow 4a^2 - 28a + 49 = 4(b-2) \Rightarrow 4a^2 - 28a + 49 = 4b - 8$ ❶
$a = 2\sqrt{b}$ (kwadrateren) $\Rightarrow a^2 = 4b \Rightarrow 4b = a^2$ ❷
❷ in ❶ $\Rightarrow 4a^2 - 28a + 49 = a^2 - 8$
$3a^2 - 28a + 57 = 0$
$D = (-28)^2 - 4 \cdot 3 \cdot 57 = 100 \Rightarrow \sqrt{D} = 10$ (hiernaast verder)

```
(-28)²-4*3*57
                100
```

$a = \frac{28+10}{6} = \frac{38}{6} = 6\frac{1}{3} \vee a = \frac{28-10}{6} = \frac{18}{6} = 3$

```
(6+1/3)²►Frac
              361/9
```

$\underline{a = 6\frac{1}{3}}$ in ❷ $\Rightarrow 4b = \frac{361}{9} \Rightarrow \underline{b = \frac{361}{36}} = 10\frac{1}{36}$ (voldoet)
$\underline{a = 3}$ in ❷ $\Rightarrow 4b = 9 \Rightarrow b = \frac{9}{4} = 2\frac{1}{4}$ (voldoet niet).

D13d $p\ln(x) = 2x$ ❷ invullen in $px^2\ln(x) = x^3 + 8$ ❶ geeft $2x \cdot x^2 = x^3 + 8 \Rightarrow 2x^3 = x^3 + 8 \Rightarrow x^3 = 8 \Rightarrow x = 2$.
$\underline{x = 2}$ in ❷ $\Rightarrow p\ln(2) = 4 \Rightarrow \underline{p = \frac{4}{\ln(2)}}$.

D14a $f(x)=\frac{x+2}{x^2-3x+2}=\frac{-3}{x-1}+\frac{4}{x-2}$ (zie de berekening hieronder) $\Rightarrow F(x)=-3\ln|x-1|+4\ln|x-2|+c.$

$f(x)=\frac{x+2}{x^2-3x+2}=\frac{x+2}{(x-1)(x-2)}=\frac{a}{x-1}+\frac{b}{x-2}=\frac{a(x-2)}{(x-1)(x-2)}+\frac{b(x-1)}{(x-1)(x-2)}=\frac{ax-2a+bx-b}{(x-1)(x-2)}=\frac{(a+b)x-2a-b}{(x-1)(x-2)}$

$\begin{cases} a+b=1 & ❶ \\ -2a-b=2 & ❷ \end{cases}$ +

$-a=3\Rightarrow a=-3$ in ❶ $\Rightarrow -3+b=1\Rightarrow b=4.$

D14b $g(x)=\frac{6x-4}{x^2-4}=\frac{4}{x+2}+\frac{2}{x-2}$ (zie de berekening hieronder) $\Rightarrow G(x)=4\ln|x+2|+2\ln|x-2|+c.$

$g(x)=\frac{6x-4}{x^2-4}=\frac{6x-4}{(x+2)(x-2)}=\frac{a}{x+2}+\frac{b}{x-2}=\frac{a(x-2)}{(x+2)(x-2)}+\frac{b(x+2)}{(x+2)(x-2)}=\frac{ax-2a+bx+2b}{(x+2)(x-2)}=\frac{(a+b)x-2a+2b}{(x-4)(x-2)}$

$\begin{cases} a+b=6 & ❶ & |2| \\ -2a+2b=-4 & ❷ & |1| \end{cases}\Rightarrow\begin{cases} 2a+2b=12 & ❸ \\ -2a+2b=-4 & ❷ \end{cases}$ +

$4b=8\Rightarrow b=2$ in ❶ $\Rightarrow a+2=6\Rightarrow a=4.$

D15 $f_a(x)=ax-x^2\Rightarrow f_a{}'(x)=a-2x$ en $g(x)=\frac{1}{2}x^2\Rightarrow g'(x)=x.$

$f_a(x)=g(x)\ \wedge\ f_a{}'(x)\cdot g'(x)=-1$

$ax-x^2=\frac{1}{2}x^2\ \wedge\ (a-2x)\cdot x=-1$

$ax=1\frac{1}{2}x^2\Rightarrow a=1\frac{1}{2}x$ ❶ $\wedge\ ax-2x^2=-1$ ❷

❶ in ❷ $\Rightarrow 1\frac{1}{2}x^2-2x^2=-1\Rightarrow -\frac{1}{2}x^2=-1\Rightarrow x^2=2\Rightarrow x=\sqrt{2}\ \vee\ x=-\sqrt{2}.$

$x=\sqrt{2}$ in ❶ $\Rightarrow a=1\frac{1}{2}\sqrt{2}$ en $x=-\sqrt{2}$ in ❶ $\Rightarrow a=-1\frac{1}{2}\sqrt{2}.$

Gemengde opgaven 14. Algebraïsche vaardigheden

G27a $\frac{x^2-5x+4}{x+1}=x+1\ (x\neq-1)$

$x^2-5x+4=(x+1)^2$

$x^2-5x+4=(x+1)(x+1)$

$x^2-5x+4=x^2+2x+1$

$-7x=-3$

$x=\frac{-3}{-7}=\frac{3}{7}.$

G27b $(\sin(x)-3)^4=(2\sin(x)-4)^4$

$\sin(x)-3=2\sin(x)-4\ \vee\ \sin(x)-3=-(2\sin(x)-4)$

$-\sin(x)=-1\ \vee\ \sin(x)-3=-2\sin(x)+4$

$\sin(x)=1\ \vee\ 3\sin(x)=7$

$x=\frac{1}{2}\pi+k\cdot2\pi\ \vee\ \sin(x)=\frac{7}{3}>1$ (geen oplossing).

G27c $\frac{e^x}{e^x+2}=\frac{e^x}{3e^x-1}\ (e^x\neq\frac{1}{3})$

$e^x(3e^x-1)=2e^x(e^x+2)$

$e^x=0$ (kan niet) $\vee\ 3e^x-1=2(e^x+2)$

$3e^x-1=2e^x+4$

$e^x=5$

$x=\ln(5).$

G27d $(2\ln(x)-1)\left(3\ln^2(x)-10\right)=4\ln(x)-2$

$(2\ln(x)-1)\left(3\ln^2(x)-10\right)=2(2\ln(x)-1)$

$2\ln(x)-1=0\ \vee\ 3\ln^2(x)-10=2$

$2\ln(x)=1\ \vee\ 3\ln^2(x)=12$

$\ln(x)=\frac{1}{2}\ \vee\ \ln^2(x)=4$

$x=e^{\frac{1}{2}}\ \vee\ \ln(x)=2\ \vee\ \ln(x)=-2$

$x=\sqrt{e}\ \vee\ x=e^2\ \vee\ x=e^{-2}$

$x=\sqrt{e}\ \vee\ x=e^2\ \vee\ x=\frac{1}{e^2}.$

G28a $y = \frac{10^x}{10^x - 1}\left(10^x - \frac{1}{10^x}\right) = \frac{10^x \cdot 10^x}{10^x - 1} - \frac{10^x}{(10^x - 1)10^x} = \frac{10^{2x}}{10^x - 1} - \frac{1}{10^x - 1} = \frac{10^{2x} - 1}{10^x - 1} = \frac{(10^x + 1)(10^x - 1)}{10^x - 1} = 10^x + 1.$

G28b $y = \frac{10 - \frac{2x}{x+3}}{5 + \frac{2}{x+3}} = \frac{10 - \frac{2x}{x+3}}{5 + \frac{2}{x+3}} \cdot \frac{x+3}{x+3} = \frac{10(x+3) - 2x}{5(x+3) + 2} = \frac{10x + 30 - 2x}{5x + 15 + 2} = \frac{8x + 30}{5x + 17}.$

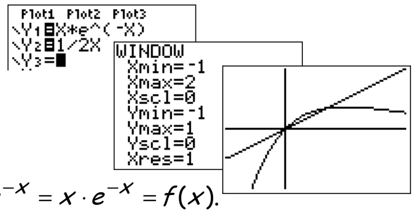
G28c $P = \frac{\frac{2}{a} - \frac{3}{a+1}}{1-a} = \frac{\frac{2}{a} - \frac{3}{a+1}}{1-a} \cdot \frac{a(a+1)}{a(a+1)} = \frac{2(a+1) - 3a}{a(a+1)(1-a)} = \frac{2a + 2 - 3a}{a(a+1)(1-a)} = \frac{2-a}{a(a+1)(1-a)}.$

G28d $P = \frac{\frac{5}{a} + \frac{6}{b}}{2b - \frac{3}{a}} = \frac{\frac{5}{a} + \frac{6}{b}}{2b - \frac{3}{a}} \cdot \frac{ab}{ab} = \frac{5b + 6a}{2ab^2 - 3b}.$

G29a $f(x) = x \cdot e^{-x} \Rightarrow f'(x) = 1 \cdot e^{-x} + x \cdot e^{-x} \cdot -1 = 1 \cdot e^{-x} - x \cdot e^{-x} = (1-x)e^{-x}.$

$f'(x) = 0 \Rightarrow (1-x)e^{-x} = 0 \Rightarrow 1 - x = 0 \Rightarrow -x = -1 \Rightarrow x = 1.$

maximum (zie een plot) $f(1) = 1 \cdot e^{-1} = e^{-1} = \frac{1}{e}.$



G29b $F(x) = (-x-1) \cdot e^{-x} \Rightarrow F'(x) = -1 \cdot e^{-x} + (-x-1) \cdot e^{-x} \cdot -1 = -e^{-x} + x \cdot e^{-x} + e^{-x} = x \cdot e^{-x} = f(x).$

G29c $f(x) = ax$

$x \cdot e^{-x} = ax$

$x = 0 \ \vee \ e^{-x} = a$

$x = 0 \ \vee \ -x = \ln(a)$

$x = 0 \ \vee \ x = -\ln(a).$

$$O(V) = \int_0^{-\ln(a)} (f(x) - ax)\,dx = \left[(-x-1)e^{-x} - \tfrac{1}{2}ax^2\right]_0^{-\ln(a)}$$

$$= (\ln(a) - 1)e^{\ln(a)} - \tfrac{1}{2}a(-\ln(a))^2 - \left(-1e^0 - 0\right)$$

$$= (\ln(a) - 1) \cdot a - \tfrac{1}{2}a \cdot \ln^2(a) + 1$$

$$= a\ln(a) - a - \tfrac{1}{2}a \cdot \ln^2(a) + 1.$$

G30a $\frac{a+b}{b+2} = \frac{3}{a}$

$a(a+b) = 3(b+2)$

$a^2 + ab = 3b + 6$

$ab - 3b = -a^2 + 6$

$b(a-3) = -a^2 + 6$

$b = \frac{-a^2 + 6}{a - 3}.$

G30b $\frac{3x+2}{x-1} = \frac{6y+1}{y+3}$

$(3x+2)(y+3) = (x-1)(6y+1)$

$3xy + 9x + 2y + 6 = 6xy + x - 6y - 1$

$-3xy + 8y = -8x - 7$

$3xy - 8y = 8x + 7$ ❶

$y(3x - 8) = 8x + 7$

$y = \frac{8x+7}{3x-8}$

Uit ❶ volgt ook $3xy - 8x = 8y + 7$

$x(3y - 8) = 8y + 7$

$x = \frac{8y+7}{3y-8}.$

G31a $f(x) = (2x+1) \cdot \sqrt{x^2+1} \Rightarrow f'(x) = 2 \cdot \sqrt{x^2+1} + (2x+1) \cdot \frac{1}{2 \cdot \sqrt{x^2+1}} \cdot 2x = 2\sqrt{x^2+1} + \frac{x(2x+1)}{\sqrt{x^2+1}}$

$= 2\sqrt{x^2+1} \cdot \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}} + \frac{x(2x+1)}{\sqrt{x^2+1}} = \frac{2(x^2+1)}{\sqrt{x^2+1}} + \frac{x(2x+1)}{\sqrt{x^2+1}} = \frac{2x^2 + 2 + 2x^2 + x}{\sqrt{x^2+1}} = \frac{4x^2 + x + 2}{\sqrt{x^2+1}}.$

G31b $G(x) = (px+q) \cdot \sqrt{x^2+1} \Rightarrow G'(x) = p \cdot \sqrt{x^2+1} + (px+q) \cdot \frac{1}{2 \cdot \sqrt{x^2+1}} \cdot 2x = p\sqrt{x^2+1} + \frac{x(px+q)}{\sqrt{x^2+1}}$

$= p\sqrt{x^2+1} \cdot \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}} + \frac{x(px+q)}{\sqrt{x^2+1}} = \frac{p(x^2+1)}{\sqrt{x^2+1}} + \frac{x(px+q)}{\sqrt{x^2+1}} = \frac{px^2 + p + px^2 + qx}{\sqrt{x^2+1}} = \frac{2px^2 + qx + p}{\sqrt{x^2+1}}.$

$G'(x) = g(x)$ geeft $2p = 2 \ \wedge \ q = 1 \ \wedge \ p = 1.$

$p = 1 \ \wedge \ q = 1.$

G31c $\int_0^{\sqrt{3}} g(x)\,dx = \left[(x+1) \cdot \sqrt{x^2+1}\right]_0^{\sqrt{3}} = (\sqrt{3}+1) \cdot \sqrt{3+1} - 1 \cdot \sqrt{1} = (\sqrt{3}+1) \cdot 2 - 1 = 2\sqrt{3} + 2 - 1 = 2\sqrt{3} + 1.$

G32a $N = 2^{3t} \cdot 3^{2t-1}$

$= 2^{3t} \cdot 3^{2t} \cdot 3^{-1}$

$= \left(2^3\right)^t \cdot \left(3^2\right)^t \cdot \frac{1}{3}$

$= \frac{1}{3} \cdot 8^t \cdot 9^t$

$= \frac{1}{3} \cdot (8 \cdot 9)^t$

$= \frac{1}{3} \cdot 72^t.$

G32b $N = \frac{3^{2t-1}}{5^{t+1}}$

$= 3^{2t-1} \cdot 5^{-t-1}$

$= 3^{2t} \cdot 3^{-1} \cdot 5^{-t} \cdot 5^{-1}$

$= \left(3^2\right)^t \cdot \frac{1}{3} \cdot \left(5^{-1}\right)^t \cdot \frac{1}{5}$

$= \frac{1}{15} \cdot 9^t \cdot \left(\frac{1}{5}\right)^t$

$= \frac{1}{15} \cdot \left(9 \cdot \frac{1}{5}\right)^t$

$= \frac{1}{15} \cdot \left(\frac{9}{5}\right)^t.$

G32c $N = 15 \cdot 3^{0{,}6t+1} = 15 \cdot 3^{0{,}6t} \cdot 3^1$

$45 \cdot 3^{0{,}6t} = N$

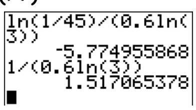
$3^{0{,}6t} = \frac{1}{45}N$

$\ln(3^{0{,}6t}) = \ln(\frac{1}{45}N)$

$0{,}6t \cdot \ln(3) = \ln(\frac{1}{45}) + \ln(N)$

$t = \frac{\ln(\frac{1}{45})}{0{,}6\ln(3)} + \frac{\ln(N)}{0{,}6\ln(3)}$

$t \approx -5{,}77 + 1{,}52\ln(N).$



G33 Stel $x_D = p$ (> 0) dan is $x_B = 3p$ (omdat $CD : DB = 1 : 2$).
$y_D = y_B$ geeft dan $f_a(p) = f_a(3p)$.
$\frac{p}{2} + \frac{a}{p} = \frac{3p}{2} + \frac{a}{3p} \Rightarrow \frac{a}{p} - \frac{a}{3p} = \frac{3p}{2} - \frac{p}{2} \Rightarrow \frac{3a}{3p} - \frac{a}{3p} = \frac{2p}{2} \Rightarrow \frac{2a}{3p} = \frac{p}{1} \Rightarrow 3p^2 = 2a \Rightarrow p^2 = \frac{2}{3}a$
$O(ABCD) = OA \cdot AB = 3p \cdot f_a(3p) = 3p \cdot f_a(p) = 3p \cdot \left(\frac{p}{2} + \frac{a}{p}\right) = \frac{3}{2}p^2 + 3a$
$\Rightarrow O(ABCD) = \frac{3}{2} \cdot \frac{2}{3}a + 3a = 4a.$

G34a $L = 80$ geeft $T = -2{,}57\ln\left(\frac{87-80}{63}\right) = -2{,}57\ln\left(\frac{7}{63}\right) = -2{,}57\ln\left(\frac{1}{9}\right) \approx 5{,}65$ (jaar).
De leeftijd is ongeveer 5 jaar en 8 maanden.

```
-2.57ln((87-80)/
63)
      5.646867164
Ans-5
      .6468671638
Ans*12
      7.762405965
```

G34b $T = 10$ geeft $10 = -2{,}57\ln\left(\frac{87-L}{63}\right)$ (algebraisch of intersect) $\Rightarrow L \approx 85{,}7$ (feet).
Dus de lengte is ongeveer $\text{Ans} \cdot 0{,}314 \approx 26{,}9$ meter.

```
10/-2.57
     -3.891050584
e^(Ans)
      .0204238778
Ans*63
        1.2867043
```

```
Ans-87
     -85.7132957
Ans*-1
      85.7132957
Ans*0.314
     26.91397485
```

G34c $T = -2{,}57\ln\left(\frac{87-L}{63}\right)$
$\frac{1}{-2{,}57}T = \ln\left(\frac{87-L}{63}\right)$
$\frac{87-L}{63} = e^{-\frac{1}{2{,}57}T}$
$87 - L = 63e^{-\frac{1}{2{,}57}T}$
$-L = -87 + 63e^{-\frac{1}{2{,}57}T}$
$L \approx 87 - 63e^{-0{,}39T}$.

```
-1/2.57
     -.3891050584
```

G35 Loodrecht snijden geeft: $f(x) = g_p(x) \wedge f'(x) \cdot g_p'(x) = -1$.
$f(x) = x^2 - 8 \Rightarrow f'(x) = 2x$ en $g_p(x) = \sqrt{2x+p} \Rightarrow g_p'(x) = \frac{1}{2 \cdot \sqrt{2x+p}} \cdot 2 = \frac{1}{\sqrt{2x+p}}$.
$x^2 - 8 = \sqrt{2x+p} \wedge 2x \cdot \frac{1}{\sqrt{2x+p}} = -1$
$\sqrt{2x+p} = x^2 - 8$ invullen in $2x \cdot \frac{1}{\sqrt{2x+p}} = -1$ geeft $2x \cdot \frac{1}{x^2-8} = -1 \Rightarrow$
$2x = -\left(x^2 - 8\right) \Rightarrow x^2 + 2x - 8 = 0 \Rightarrow (x+4)(x-2) = 0 \Rightarrow x = -4 \vee x = 2.$
$x = -4$ en $\sqrt{2x+p} = x^2 - 8$ geeft $\sqrt{-8+p} = 8 \Rightarrow -8 + p = 64 \Rightarrow p = 72.$
($x = 2$ en $\sqrt{2x+p} = x^2 - 8$ geeft $\sqrt{4+p} = -4$ heeft geen oplossing)

G36a $b = 0{,}018$ en $S = 58 \Rightarrow 58 = \frac{a + 0{,}018}{a \cdot 0{,}018} \Rightarrow 58 \cdot 0{,}018a = a + 0{,}018 \Rightarrow (1{,}044 - 1)a = 0{,}018 \Rightarrow a \approx 0{,}41.$
$b = 0{,}018$ en $S = 63 \Rightarrow 63 = \frac{a + 0{,}018}{a \cdot 0{,}018} \Rightarrow 63 \cdot 0{,}018a = a + 0{,}018 \Rightarrow (1{,}044 - 1)a = 0{,}018 \Rightarrow a \approx 0{,}13.$
Het rechteroog kan tussen 0,13 meter en 0,41 meter scherp zien.

```
58*0.018
            1.044
Ans-1
             .044
0.018/Ans
      .4090909091
```

```
63*0.018
            1.134
Ans-1
             .134
0.018/Ans
      .1343283582
```

G36b $S = \frac{a+b}{a \cdot b} \Rightarrow abS = a + b \Rightarrow abS - a = b \Rightarrow a(bS - 1) = b \Rightarrow a = \frac{b}{bS - 1}$.

G36c $b = 0{,}017$ en $a = 0{,}15 \Rightarrow S = \frac{0{,}15 + 0{,}017}{0{,}15 \cdot 0{,}017} \approx 65{,}49.$
$b = 0{,}017 \Rightarrow S = \frac{a + 0{,}017}{a \cdot 0{,}017} = \frac{a}{a \cdot 0{,}017} + \frac{0{,}017}{a \cdot 0{,}017} = \frac{1}{0{,}017} + \frac{1}{a}.$
Voor grote waarden van a nadert $S = \frac{1}{0{,}017} + \frac{1}{a}$ naar $S = \frac{1}{0{,}017} \approx 58{,}82.$
Dus S kan de waarden 59 tot en met 65 aannemen.

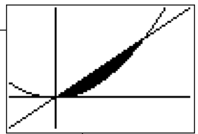
```
(0.15+0.017)/(0.
15*0.017)
      65.49019608
1/0.017
      58.82352941
```

G37a $\frac{1}{2}x^2 = x \Rightarrow x = 0 \vee \frac{1}{2}x = 1 \Rightarrow x = 0 \vee x = 2.$
$I = I_{kegel} - \int_0^2 \pi(\tfrac{1}{2}x^2)^2\,dx = \frac{1}{3}\pi \cdot 2^2 \cdot 2 - \int_0^2 \frac{1}{4}\pi x^4\,dx = \frac{8}{3}\pi - \left[\frac{1}{20}\pi x^5\right]_0^2$
$= \frac{8}{3}\pi - \left(\frac{1}{20}\pi \cdot 2^5 - 0\right) = \frac{8}{3}\pi - \frac{32}{20}\pi = 1\frac{1}{15}\pi.$

```
8/3-32/20▸Frac
            16/15
```

```
Plot1 Plot2 Plot3
\Y1■1/2X²
\Y2■X
\Y3=
```

```
WINDOW
 Xmin=-1
 Xmax=3
 Xscl=0
 Ymin=-1
 Ymax=3
 Yscl=0
 Xres=1
```



G37b Omdat $OP_nQ_nR_n$ een vierkant is, geldt: $\frac{1}{n}x^2 = x \Rightarrow x = 0 \vee \frac{1}{n}x = 1 \Rightarrow x = 0$ (in de oorsprong) $\vee\ x = n$ (in P_n).
De richtingscoëfficiënt van de raaklijn in P_n is $\left[\frac{dy}{dx}\right]_{x=n} = \left[\frac{2}{n} \cdot x\right]_{x=n} = \frac{2}{n} \cdot n = 2$ en dit is onafhankelijk van n.

G38a $100 = 200 - 180 \cdot e^{-0{,}29t} \Rightarrow 180 \cdot e^{-0{,}29t} = 100 \Rightarrow e^{-0{,}29t} = \frac{100}{180} \Rightarrow -0{,}29t = \ln(\frac{100}{180}) \Rightarrow t \approx 2{,}027.$
Het opwarmen wordt gestopt na 2 uur en 2 minuten, dus om 17:02.

```
100/180
      .5555555556
ln(Ans)
     -.5877866649
Ans/-0.29
      2.026850569
(Ans-2)*60
```

G38b $S(t) = 200 - 180 \cdot e^{-0{,}29t} \Rightarrow S'(t) = -180 \cdot e^{-0{,}29t} \cdot -0{,}29 = 52{,}2 \cdot e^{-0{,}29t}$.
Om 16:00 is $t = 1$ en $S'(1) = 52{,}2 \cdot e^{-0{,}29} \approx 39{,}06$ (°C/uur).
Dit is een snelheid van ongeveer 0,7 graden Celcius per minuut.

```
-180*-0.29
             52.2
Ans*e^(-0.29)
      39.05935823
Ans/60
      .6509893038
```

G38c $S = 200 - 180 \cdot e^{-0{,}29t}$
$180 \cdot e^{-0{,}29t} = 200 - S$
$e^{-0{,}29t} = \frac{200 - S}{180}$
$-0{,}29t = \ln\left(\frac{200 - S}{180}\right)$
$t = -\frac{1}{0{,}29} \cdot \ln\left(\frac{200 - S}{180}\right) \approx -3{,}45 \cdot \ln\left(\frac{200 - S}{180}\right).$

```
-1/0.29
     -3.448275862
```